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Warehousing and distribution network design from a Third-Party Logistics (3PL) company perspective

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Abstract

In this thesis we deal with the problem of optimizing the logistics network of a Third-Party Logistics (3PL) company. The goal is to minimize the cost of storage and transport operations, in the case of multiple warehouses, multiple suppliers, multiple clients, multiple products and multiple types of transportation vehicles.

We model this problem by a new Mixed Integer Linear Program (MILP). The related decisions include selection of (a) the warehouse(s) to store each product (SKU), (b) the inventory level per SKU per warehouse, (c) the warehouse(s) to serve each customer and (d) the appropriate vehicles to transport the products from the suppliers to the warehouses, and from the latter to the final customers.

We implemented the model in Python Pulp and solved it using Gurobi Optimizer 9.1.2. Multiple validation tests were performed to confirm the correct structure, and completeness of the model. Subsequently the model was applied in case study of a 3PL company in Thessaloniki Greece. The company's network consists of: (a) three warehouses located in the industrial area of Thessaloniki, (b) 23 suppliers throughout Greece that ship one or more products, 53 customers, the majority of which are located in northern Greece. The company manages 41 dry products classified into 13 product families. All operations are performed in pallets. The transport fleet comprises commercial vehicles (Δημόσιας Χρήσης). All information was provided directly from the company.

The proposed method was fully capable to model this complex practical environment, Furthermore, the results obtained were very encouraging, since overall warehousing and distribution costs were lowered by 10.84% compared to the way the company operates currently.

Εκτενής περίληψη (Summary in Greek)

Περιγραφή προβλήματος

Στην παρούσα διπλωματική εργασία αναπτύσσεται νέο μαθηματικό μοντέλο Μικτού Ακέραιου Γραμμικού Προγραμματισμού (ΜΑΓΠ) με σκοπό την ελαχιστοποίηση του αποθηκευτικού και μεταφορικού κόστους εταιρείας Third-Party Logistics (3PL). Το μοντέλο καλύπτει τις σημαντικότερες αποφάσεις που σχετίζονται με τον αποτελεσματικό σχεδιασμό δικτύου εφοδιαστικής αλυσίδας λαμβάνοντας υπόψη πολλαπλές χρονικές περιόδους, προϊόντα, αποθήκες, πελάτες, προμηθευτές και τύπους οχημάτων μεταφοράς. Οι σχετικές αποφάσεις ανά χρονική περίοδο είναι οι εξής:

(α) Ποιά αποθήκη (ή αποθήκες) θα χρησιμοποιηθεί (ούν) για τροφοδοσία κάθε προϊόντος από κάθε προμηθευτή;

(β) Ποιο είναι το επίπεδο αποθεματοποίησης ανά προϊόν και ανά αποθήκη;

(γ) Ποιά αποθήκη θα αποστέλλει ποια προϊόντα σε κάθε πελάτη;

(δ) Ποιοι τύποι και πόσα οχήματα θα μεταφέρουν τα προϊόντα από τους προμηθευτές στις αποθήκες και από εκεί στους τελικούς πελάτες;

Τα αποθηκευτικά κόστη που λαμβάνονται υπόψη, είναι αυτά της εκφόρτωσης, αποθήκευσης αποθέματος, συλλογής και φόρτωσης. Τα μεταφορικά κόστη αποτελούνται από το κόστος μεταφοράς από τους προμηθευτές ως τις αποθήκες και από τις αποθήκες στους τελικούς πελάτες.

Συγκεκριμένα, το πρόβλημα που αναλύεται αφορά τις εταιρείες 3PL που βελτιστοποιούν το δίκτυο χωροθέτησης-αποθήκευσης-μεταφοράς εντός καθορισμένου χρονικού ορίζοντα με σκοπό να εξυπηρετήσουν τη ζήτηση. Στο πρόβλημα λαμβάνονται υπόψη οι ακόλουθες οντότητες και δραστηριότητες: (α) πελατειακή ζήτηση, (β) προμηθευτές, (γ) αποθήκες, (δ) μεταφορικό σύστημα.

Πελάτες και ζήτηση

Οι πελάτες μπορεί να είναι χονδρέμποροι, λιανέμποροι, καταστήματα κλπ. Οι γεωγραφικές θέσεις τους θεωρούνται γνωστές. Η ζήτηση ανά κωδικό και πελάτη εκφράζεται σε παλέτες ανά χρονική περίοδο.

Προμηθευτές

Οι προμηθευτές παράγουν ή εμπορεύονται προϊόντα διαφορετικών κωδικών. Κάθε προμηθευτής μπορεί να προμηθεύσει ποσότητες μέχρι ένα προκαθορισμένο όριο ανά κωδικό και ανά χρονική περίοδο.

Αποθήκες

Οι αποθήκες παραλαμβάνουν παλετοποιημένα προϊόντα από τους προμηθευτές. Θεωρούμε πως κάθε αποθήκη έχει ένα ανώτατο όριο χωρητικότητας παλετών. Ο αριθμός των προϊόντων που μεταφέρονται ανά παλέτα διαφέρει ανάλογα με τον κωδικό του προϊόντος. Στα πλαίσια διαχείρισης του αποθέματος, κάθε προϊόν σχετίζεται με ένα ελάχιστο επίπεδο αποθέματος ασφαλείας.

Μεταφορά

Πολλαπλοί τύποι οχημάτων και συνδυασμοί αυτών μπορούν να χρησιμοποιηθούν για τη μεταφορά από τους προμηθευτές στις αποθήκες και από τις αποθήκες στους πελάτες. Η θεώρηση αυτή είναι απαραίτητη σε περιβάλλον 3PL που συνήθως χρησιμοποιούνται οχήματα ΔΧ (Δημοσίας χρήσης).

Προτεινόμενο μαθηματικό μοντέλο

Πριν από τον καθορισμό της αντικειμενικής συνάρτησης και των σχετικών περιορισμών, ορίζουμε τα απαραίτητα στοιχεία του μαθηματικού μοντέλου τα οποία είναι οι κόμβοι, οι συνδέσεις-τόξα, οι παράμετροι και οι μεταβλητές απόφασης.

Δίκτυο

Κόμβοι	S: *Υφιστάμενοι προμηθευτές*
	W: *Υφιστάμενες αποθήκες*
	C: *Υφιστάμενοι πελάτες*
	$N = S \cup W \cup C$
Συνδέσεις-Τόξα	$A_f = \{(i,j) \mid i \in S, j \in W\} \cup \{(i,j) \mid i \in W, j \in C\}$: *Σύνολο συνδέσεων-τόξων μεταφοράς* παλετών
Προϊόντα	$p \in P$ *όπου* P *το σύνολο των προϊόντων*
Χρονικές περίοδοι	T: *Υφιστάμενες χρονικοί περίοδοι*

Παράμετροι

Αφορά τα ακόλουθα σύνολα: (α) κόστη: παραλαβές, συλλογή (picking) και αποστολές, απόθεμα, μεταφορές, (β) χωρητικότητα-δυναμικότητα: αποθηκών, προμηθευτών, οχημάτων, οικογενειών προϊόντων, (γ) ζήτηση προϊόντων σε παλέτες και (δ) το επίπεδο αποθέματος ασφαλείας για κάθε προϊόν και το απόθεμα της αρχικής χρονικής περιόδου.

Κόστη	
Κόστος παραλαβής παλέτας	R_w: *Κόστος παραλαβής και τοποθέτησης παλετών στην αποθήκη* $w \in W$
Κόστος συλλογής (picking) & αποστολής παλέτας	S_w: *Κόστος συλλογής (picking) και αποστολής παλετών στην αποθήκη* $w \in W$
Κόστος αποθεματοποίησης	N_{pw}: *Κόστος αποθεματοποίησης (μιας παλέτας) προϊόντος* $p \in P$ *στην αποθήκη* $w \in W$
Κόστος μεταφοράς	E_{ij}^{v}: *Κόστος μεταφοράς με συνδυασμό τύπου οχήματος* $v \in V$ *από τον κόμβο* i *στον κόμβο* j *όπου* $(i, j) \in A$

Χωρητικότητα - Δυναμικότητα

Χωρητικότητα αποθήκης	Q_w: *Χωρητικότητα αποθήκης* $w \in W$ *(σε παλέτες)*
Δυναμικότητα προμηθευτή	Q_{spt}: *Δυναμικότητα του προμηθευτή* $s \in S$ *για το προϊόν* $p \in P$ *τη χρονική περίοδο* $t \in T$
Χωρητικότητα τύπου μεταφοράς	M^v: *Συνολική χωρητικότητα συνδυασμού οχημάτων* $v \in V$
Κατώτερο όριο συντελεστή φόρτωσης οχήματος	F^v: *Κάτω όριο συντελεστή φόρτωσης συνδυασμού οχημάτων* $v \in V$ *(μπορεί να είναι μηδέν)*

Προϊόντα & Ζήτηση

Τύποι προϊόντων	$P^s \subseteq P$: *Προϊόντα που παρέχονται από τον προμηθευτή* $s \in S$
Ζήτηση	D_{ptc}: *Ζήτηση προϊόντος* $p \in P$ *την περίοδο* $t \in T$ *για τον πελάτη* $c \in C$

Παράμετροι αποθέματος

Απόθεμα ασφαλείας	L_p: *Απόθεμα ασφαλείας του προϊόντος* $p \in P$ *(in pallets). Το απόθεμα αυτό αφορά όλες τις αποθήκες*

Αρχικό απόθεμα	i_{p0w}: *Αρχικό απόθεμα του προϊόντος* $p \in P$, *στην αποθήκη* $w \in W$ *τη στιγμή* $t = 0$

Τύποι μεταφορικών μέσων

	V: *Σύνολο των διαθέσιμων οχημάτων και των πιθανών συνδυασμών τους π.χ.*
	v = *1 όχημα van χωρητικότητας 7 παλετών* v = *2 φορτηγό όχημα χωρητικότητας 18 παλετών* v = *3 φορτηγό όχημα χωρητικότητας 33 παλετών* v = *4 συνδυασμός δύο οχημάτων χωρητικότητας 7-πάλετων* v = *5 συνδυασμός 7-πάλετου οχήματος και 18-πάλετου οχήματος* v = *6 συνδυασμός 7-πάλετου οχήματος και 33-πάλετου οχήματος*v = *7 συνδυασμός δύο 18-πάλετων οχημάτων* $v = 8$ *συνδυασμός 18-πάλετου οχήματος και 33-πάλετου οχήματος* $v = 9$ *συνδυασμός δύο 33-πάλετων οχημάτων* $v = 10$ *συνδυασμός τριών 33-πάλετων οχημάτων* $v = 11$ *συνδυασμός τεσσάρων 33- πάλετων οχημάτων* $v = 12$ *συνδυασμός πέντε 33- πάλετων οχημάτων* $v = 13$ *συνδυασμός έξι 33- πάλετων οχημάτων* $v = 14$ *συνδυασμός επτά 33- πάλετων οχημάτων* $v = 15$ *συνδυασμός οκτώ 33- πάλετων οχημάτων* $v = 16$ *συνδυασμός εννιά 33- πάλετων οχημάτων* $v = 17$ *συνδυασμός δέκα 33- πάλετων οχημάτων* $v = 18$ *συνδυασμός έντεκα 33- πάλετων οχημάτων* $v = 19$ *συνδυασμός δώδεκα 33- πάλετων οχημάτων* $v = 20$ *συνδυασμός δεκατριών 33- πάλετων οχημάτων*

Μεταβλητές απόφασης

Μεταφερόμενες ποσότητες	x_{ptij}^{v}: *Ο αριθμός παλετών προϊόντος* $p \in P$ *που μεταφέρονται με τον συνδυασμό οχημάτων* $v \in V$ *την περίοδο (π.χ.. μία ημέρα)* $t \in T$ *από τον κόμβο i στον κόμβο j όπου* $(i, j) \in A_f$
Απόθεμα	i_{ptw}: *Απόθεμα (σε παλέτες) του προϊόντος* $p \in P$ *στο τέλος της χρονικής περιόδου (ημέρας)* $t \in T$ *στην αποθήκη* $w \in W$
Χρήση οχημάτων	m_{tij}^{v}: *Λαμβάνει την τιμή 1, αν ο συνδυασμός οχημάτων* $v \in V$ *χρησιμοποιείται τη χρονική στιγμή* $t \in T$ *για να διασχίσει το τόξο* $(i, j) \in A$, *διαφορετικά, λαμβάνει την τιμή 0*

Η μαθηματική διατύπωση του προτεινόμενου μοντέλου παρουσιάζεται παρακάτω.

Αντικειμενική συνάρτηση

Στόχος της αντικειμενικής συνάρτησης είναι η ελαχιστοποίηση του συνολικού κόστους των λειτουργιών αποθήκευσης και μεταφοράς μιας εταιρείας 3PL στη διάρκεια του υπό μελέτη χρονικού ορίζοντα. Το συνολικό κόστος αποτελείται από τα παρακάτω επιμέρους κόστη:

(α) το κόστος αποθεματοποίησης: $(\sum_{t\in T}\sum_{p\in P}\sum_{w\in W}(N_{pw} i_{pwt}))$,

(β) Το κόστος λειτουργιών της αποθήκης:

$(\sum_{t\in T}\sum_{p\in P}\sum_{s\in S}\sum_{w\in W}\sum_{v\in V} R_w x^v_{ptsw} + \sum_{t\in T}\sum_{p\in P}\sum_{w\in W}\sum_{c\in C}\sum_{v\in V} S_w x^v_{ptwc})$,

(γ) το κόστος μεταφοράς: $(\sum_{t\in T}\sum_{v\in V}\sum_{(i,j)\in A} E^v_{ij} m^v_{tij})$.

Η μαθηματική διατύπωση της αντικειμενικής συνάρτησης έχει ως εξής:

$$min \sum_{t\in T}\sum_{p\in P}\sum_{w\in W} N_{pw} i_{pwt} + \sum_{t\in T}\sum_{p\in P}\sum_{s\in S}\sum_{w\in W}\sum_{v\in V} R_w x^v_{ptsw} + \sum_{t\in T}\sum_{p\in P}\sum_{w\in W}\sum_{c\in C}\sum_{v\in V} S_w x^v_{ptwc} + \sum_{t\in T}\sum_{v\in V}\sum_{(i,j)\in A} E^v_{ij} m^v_{tij} \quad (3.1)$$

Οι περιορισμοί του προτεινόμενου μοντέλου είναι οι εξής:

Πελατειακή ζήτηση

Ο περιορισμός (3.2) εξασφαλίζει ότι η ζήτηση κάθε πελάτη για κάθε προϊόν και για κάθε περίοδο ικανοποιείται.

$$\sum_{w\in W}\sum_{v\in V} x^v_{ptwc} \geq D_{ptc}, \quad p \in P, c \in C, t \in T \quad (3.2)$$

Μεταφερόμενες ποσότητες

Ο περιορισμός (3.3) διασφαλίζει πως οι μεταφερόμενες ποσότητες στο τόξο $(i,j) \in A_f$, πρέπει να είναι μεγαλύτερες ή ίσες με το κατώτατο όριο φόρτωσης F^v του οχήματος (ή συνδυασμού οχημάτων) $v \in V$. Ο περιορισμός (3.4) εξασφαλίζει πως οι μεταφερόμενες ποσότητες είναι μικρότερες ή ίσες με τη συνολική χωρητικότητα M^v του οχήματος (ή συνδυασμού οχημάτων $v \in V$, στην περίοδο $t \in T$.

$$\sum_{p \in P} x_{ptij}^{v} \geq F^{v} M^{v} m_{tij}^{v}, \quad (i,j) \in A_f, t \in T, v \in V \tag{3.3}$$

$$\sum_{p \in P} x_{ptij}^{v} \leq M^{v} m_{tij}^{v}, \quad (i,j) \in A_f, t \in T, v \in V \tag{3.4}$$

Απόθεμα

Ο περιορισμός (3.5) ορίζει την εξέλιξη του αποθέματος i_{ptw} στις αποθήκες $w \in W$. Ο περιορισμός (3.6) εξασφαλίζει πως το συνολικό απόθεμα του προϊόντος $p \in P$ σε όλες τις αποθήκες $w \in W$, είναι μεγαλύτερο ή ίσο με το απόθεμα ασφαλείας L_p του προϊόντος $p \in P$ τη χρονική περίοδο $t \in T$.

$$i_{ptw} = i_{p(t-1)w} + \sum_{s \in S} \sum_{v \in V} x_{ptsw}^{v} - \sum_{c \in C} \sum_{v \in V} x_{ptwc}^{v}, \quad p \in P, w \in W, t \in T \tag{3.5}$$

$$\sum_{w \in W} i_{ptw} \geq L_p, \quad p \in P, t \in T \tag{3.6}$$

Χωρητικότητα αποθήκης

Ο περιορισμός (3.7) διασφαλίζει πως το συνολικό απόθεμα όλων των προϊόντων στην αποθήκη $w \in W$ είναι μικρότερο από ή ίσο με τη συνολική χωρητικότητα Q_w της αποθήκης (σε παλέτες) για κάθε περίοδο $t \in T$.

$$\sum_{p \in P} i_{ptw} \leq Q_w, \quad \in W, t \in T \tag{3.7}$$

Δυναμικότητα προμηθευτή

Ο περιορισμός (3.8) διασφαλίζει πως οι συνολικές μεταφερόμενες ποσότητες x_{ptij}^{v} του προϊόντος $p \in P$ από τον προμηθευτή $s \in S$ πρέπει να είναι μικρότερες ή ίσες της χωρητικότητας Q_{spt} του προμηθευτή $s \in S$ (σε παλέτες) του προϊόντος $p \in P$ τη χρονική περίοδο $t \in T$.

$$\sum_{v \in V} \sum_{w \in W} x_{ptij}^{v} \leq Q_{spt}, \quad p \in P, t \in T, s \in S \tag{3.8}$$

Μεταβλητές απόφασης

Ο περιορισμός (3.9) ορίζει τη χρήση του οχήματος m_{tij}^{v} να είναι δυαδική, λαμβάνοντας την τιμή 1, αν ο συνδυασμός οχημάτων $v \in V$ χρησιμοποιείται στο τόξο $(i,j) \in A$ τη χρονική περίοδο $t \in T$ και 0 αν όχι. Ο περιορισμός (3.10) ορίζει πως οι μεταφερόμενες ποσότητες x_{ptij}^{v} είναι θετικοί πραγματικοί αριθμοί. Ο περιορισμός (3.11) διασφαλίζει πως το απόθεμα i_{ptw} είναι πραγματικός θετικός αριθμός.

$$m_{tij}^{v} \in \{0,1\}, v \in V, (i,j) \in A \tag{3.9}$$

$$x_{ptij}^{v} \geq 0, v \in V, t \in T, (i,j) \in A_f, \tag{3.10}$$

$$i_{ptw} \geq 0, \qquad p \in P, t \in T, w \in W \tag{3.11}$$

Υλοποίηση και επαλήθευση του μοντέλου

Το παραπάνω μοντέλο αναπτύχθηκε σε προγραμματιστικό περιβάλλον Python PuLP και επιλύθηκε με τη χρήση του λογισμικού βελτιστοποίησης Gurobi Optimizer 9.1.2 (Gurobi Optimization, 2021) σε υπολογιστή με επεξεργαστή Intel Core i7, συχνότητας λειτουργίας 3,4GHz και με μνήμη 8GB.

Μια σειρά επτά δοκιμών πραγματοποιήθηκε προκειμένου να επικυρωθεί η ορθότητα του προτεινόμενου μοντέλου. Αυτές οι δοκιμές εξέτασαν διάφορες απλές περιπτώσεις που περιελάμβαναν δύο προμηθευτές, δύο αποθήκες, δύο πελάτες, δύο προϊόντα και τρία οχήματα με διάφορες χωρητικότητες, σε δύο χρονικές περιόδους (δύο ημέρες). Το μοντέλο και η υλοποίησή του τελειοποιήθηκαν ώστε το σύνολο των δοκιμών να επαληθευτούν πλήρως.

- Δοκιμή (α) - Επικύρωσε τον περιορισμό της δυναμικότητας των προμηθευτών και του αποθέματος ασφαλείας.
- Δοκιμή (β) - Επικύρωσε τον περιορισμό του συντελεστή φόρτωσης των οχημάτων (lower bound).
- Δοκιμή (γ) - Επικύρωσε τον περιορισμό αποθέματος και χωρητικότητας της αποθήκης.
- Δοκιμή (δ) - Επικύρωσε τον περιορισμό χωρητικότητας των οχημάτων.

- Δοκιμή (ε) - Επικύρωσε τον περιορισμό της ζήτησης πελατών και την επιλογή βέλτιστης διαδρομής.
- Δοκιμή (στ) - Επικύρωσε τον περιορισμό της ζήτησης των πελατών και τον περιορισμό της δυναμικότητας των προμηθευτών.
- Δοκιμή (η) - Επικύρωσε τον περιορισμό της ζήτησης των πελατών και του αποθέματος ασφαλείας.

Μελέτη Περίπτωσης

Η μελέτη περίπτωσης εστίασε σε εταιρεία Third-Party Logistics (3PL) που δραστηριοποιείται σε όλη την Ελλάδα, με βάσεις την Αθήνα, τη Θεσσαλονίκη και την Πάτρα. Σε όλη τη χώρα η εταιρία διαχειρίζεται συνολικά 30 αποθήκες συνολικής επιφάνειας άνω των 130.000 m^2. Η εταιρεία διαχειρίζεται παλέτες με προϊόντα ξηρού φορτίου που εξυπηρετεί τις ανάγκες των πελατών της προσπαθώντας να ελαχιστοποιήσει το αποθηκευτικό και μεταφορικό κόστος. Η μελέτη περίπτωσης επικεντρώνεται στις αποθήκες της περιοχής της Σίνδου Θεσσαλονίκης.

Το δίκτυο εφοδιαστικής αλυσίδας που αναλύθηκε, αποτελείται από: (α) 23 προμηθευτές σε όλη την Ελλάδα που προμηθεύουν 41 προϊόντα, (β) 3 αποθήκες στη ΒΙ.ΠΕ. Θεσσαλονίκης και (γ) 53 πελάτες σε όλη την Ελλάδα, οι περισσότεροι στη Βόρεια Ελλάδα.

Η εταιρεία 3PL παραλαμβάνει από τους προμηθευτές των πελατών της, αποθηκεύει τα προϊόντα και εφοδιάζει τους πελάτες των πελατών της ανάλογα με τη ζήτηση. Ένας προμηθευτής μπορεί να προμηθεύσει έναν ή περισσότερους κωδικούς προϊόντων. Για κάθε προϊόν διατηρείται απόθεμα ασφαλείας. Στις αποθήκες τα προϊόντα αποθηκεύονται σε ράφια back-to-back, εκτός από την τρίτη και μικρότερη αποθήκη στην οποία τα προϊόντα αποθηκεύονται στο δάπεδο. Για τις μεταφορές εντός των αποθηκών, χρησιμοποιούνται ηλεκτρικά παλετοφόρα οχήματα. Τα δρομολόγια μεταξύ προμηθευτών και αποθηκών εξυπηρετούνται αποκλειστικά από οχήματα που παρέχονται από τους προμηθευτές. Τα δρομολόγια μεταξύ αποθηκών και πελατών εξυπηρετούνται από ΔΧ φορτηγά οχήματα με βάση τις ανάγκες της εταιρείας 3PL.

Η βελτιστοποίηση (to-be) του δικτύου αντιστοιχεί σε μείωση του συνολικού κόστους μεταφοράς και αποθήκευσης κατά 10.84% σε σύγκριση με τα υφιστάμενα κόστη. Η μεγαλύτερη διαφορά σχετίζεται με τα κόστη παραλαβής, συλλογής και αποστολής όπως παρουσιάζεται στον Πίνακα 1. Μικρή αύξηση παρατηρείται στο κόστος αποθεματοποίησης και στο κόστος μεταφοράς από τις αποθήκες στους πελάτες, διότι το απόθεμα κατανέμεται σε αποθήκες με χαμηλότερα λειτουργικά κόστη. Τα κόστη μεταφοράς δεν επηρεάζονται σημαντικά καθότι οι τρεις αποθήκες γειτνιάζουν γεωγραφικά.

Πίνακας 1. Σύγκριση του κόστους του μοντέλου βελτιστοποίησης to-be που προκύπτει από προτεινόμενο μοντέλο ΜΑΓΠ έναντι του υφιστάμενου και βελτιστοποιημένου as-is

Κόστη	**Κόστος (€/εβδομάδα)**			**Διαφορά Βελτιστοποιημένο as-is - to-be (%)**	**Διαφορά Υφιστάμενο as-is - to-be (%)**
	Υφιστάμενο Μοντέλο as-is	**Βελτι/μένο Μοντέλο as-is**	**Μοντέλο to-be**		
Παραλαβών	5,817.60	5,817.60	4,443.20	-23.62%	-23.62%
Picking & Αποστολών	8,534.80	8,534.80	6,996.00	-18.02%	-18.02%
Αποθεματοποίησης	10,279.98	10,279.98	10,312.42	0.31%	0.31%
Μεταφορών προς αποθήκες	36,111.09	32,444.92	31,269.86	-3.62%	-14,59%
Μεταφορών από αποθήκες	27,134.30	25,788.16	25,843.95	0.21%	-5,02%
Σύνολο	**87,877.86**	**82,865.46**	**78,865.23**	**-4.82%**	**-10.84%**

Συμπεράσματα

Το προτεινόμενο πρωτότυπο μοντέλο αποτυπώνει πλήρως την πολυπλοκότητα του κυκλώματος μεταφοράς, αποθήκευσης και διανομής εταιρίας 3PL. Η ορθότητα της υλοποίησής του επαληθεύτηκε μέσω πολλαπλών δοκιμών. Το μοντέλο εφαρμόστηκε με απόλυτη επιτυχία σε σημαντική μελέτη περίπτωσης εταιρίας 3PL με εγκαταστάσεις στη Θεσσαλονίκη που τροφοδοτεί τη Βόρεια Ελλάδα. Τα αποτελέσματα που προέκυψαν ήταν πολύ ενθαρρυντικά, καθώς το συνολικό κόστος αποθήκευσης και διανομής μειώθηκε κατά 10.84% σε σύγκριση με τον τρόπο λειτουργίας της εταιρείας σήμερα.

Table of Contents

List of Figures

List of Tables

Chapter 1. Thesis scope

A supply chain is a system that consists of people and organisational structures, material, material transformation activities, information and resources used in fulfilling the demand for a product or service by a customer (Simchi-Levi, et al., 2017). The world is connected through supply chains and, thus, supply chains have become the prime mover of world trade, which is, in turn, is a pillar of global economy. This is evident from the share of trade in global Gross Domestic Product (GDP) which has increased from 39.2% in 1990 to 58.2% in 2019 (World Bank, 2020).

Top global third-Party Logistics (3PL) companies (i.e. DHL, Kuehne & Nagel, DB Schenker) have developed interlinked and complex global supply chains that comprise a multiplicity of suppliers, warehouses, distribution centers and business partners distributed across a global footprint. According to various reports and studies, the best companies are the ones which design efficient supply chains aiming at cost and customer service improvement (Saddikuti, et al., 2020). The design and management of a supply chain network in today's competitive business environment is an important challenge (Reeve & Srinivasan, 2005).

Distribution network optimization is one of the most significant factors of productivity and profitability of a supply chain (Forouzanfar & Tavakkoli-Moghaddam, 2012). The three major problems that have been typically taken into account in designing a distribution network are related to location-inventory-routing and more specifically are: (a) the location-allocation problem, (b) the vehicle routing problem and (c) the inventory problem (Dehghani, et al., 2017). Determining the locations and capacities of warehouses and distribution centers and specifying the volume of the transferred product from suppliers to the final customers through the distribution centers are important decisions in supply chain design (Mousavi, et al., 2015). Due to the importance and the complexity of distribution network design, significant research work has been conducted in this area, especially concerning the relationship of costs with the architecture of the distribution network.

In this Thesis, we study the optimization of the distribution network from a third-Party Logistics (3PL) company perspective, focusing on the location-inventory-routing aspects. To do so, we developed a novel Mixed Integer Linear Programming (MILP) model that covers the most significant decisions involved in efficient re-design of a supply chain network considering multiplicity of time periods, products, warehouses, customers, suppliers, transportation types. The model is able to support the designer to make the following decisions for each time period of the planning horizon:

(a) Which warehouse should be served by each supplier?
(b) Which warehouse will be selected to store the products and at what level (inventory)?
(c) Which warehouse should serve each customer?
(d) Which vehicle would be suitable to transfer the products from the suppliers to the final customers?

Warehousing costs include receiving, inventory, picking and shipping costs. Transportation costs include both the costs of transporting the goods from the suppliers to the warehouses and the corresponding costs from the warehouses to the final customers.

An important part of this Thesis is the application of the proposed model in an actual case study. Through this case study, our goal is to validate that the proposed model is all inclusive and complete, as well as capable of optimizing complex networks of actual 3PL companies.

The remainder of the Thesis is organized as follows:

Chapter 2 discusses the motivation of this Thesis, and the recent research results in related areas. The literature review focuses on location-inventory-routing aspects of the general supply chain network design problem. Chapter 2 also highlights the research gaps and outlines the contributions of this work.

Chapter 3 presents the novel mathematical model proposed to optimize the warehousing and transportation network from a third-Party Logistics (3PL) company perspective. Its objective is to minimize the warehousing and transportation costs over the design horizon. Appropriate constraints model the complex relationships among the links of the supply chain. To ensure proper modelling, a series of validation test cases were conducted and are presented in this Chapter.

Chapter 4 applies the model of Chapter 3 in a significant case study. The Chapter 4 presents the case, the data collection process, and the exact solutions obtained by a commercial solver. The resulting optimal network is compared to the current network of the company.

Finally, Chapter 5 summarizes the main findings of the current research, presents the conclusions of the work, and provides recommendations for further research.

Chapter 2. Literature background in location-inventory-routing problems

Prior to focusing on the research problem addressed in this thesis, this Chapter reviews the research state-of-the-art on the location-inventory-routing problems, identifying research gaps and setting the research goals.

Several studies in this area have focused on topics related to the current thesis. Hamedani et al. (2013) examined a location-inventory problem in a three-level supply chain network under uncertainty. Their single time period model addresses the design of a three-level supply chain network that includes multiple suppliers, distribution centers, customers, products, scenarios and a single transport model. The model is formulated as a multi-objective mixed-integer nonlinear program that minimizes the expected total cost and the cost variance of the network including the transportation, ordering, purchasing, holding, shortage, opening facility costs. Using the Lingo 9 software, they solved small problems with the Epsilon Constraint Method, while for larger ones they used a Multi-Objective Particle Swarm Optimization (MOPSO) meta-heuristic algorithm. This work does not include transportation type decisions among the key supply chain network decisions.

Forouzanfar and Tavakkoli-Moghaddam (2012) addressed a problem of designing a multi-echelon supply chain network to determine the number of the distribution centres, their locations, capacities, service and lead times and also the routes among suppliers, distribution centres and customers encompassing one product and a single transport mode. They presented a mathematical model that minimizes the establishment costs of distribution centres, the safety stock cost considering uncertainty in customer demand, the inventory ordering and holding cost and also the transportation cost among all entities. They formulated this problem using a Mix Integer Non-Linear Programming model (MINLP), and solved small-sized problems using General Algebraic Modeling System software (GAMS) and larger-sized problems through a new genetic algorithm.

Sainathuni et al. (2014) presented a Non-Linear Integer Programming model (NLIP) that considers multiple suppliers, customers, products, time-periods, and a single warehouse. They also modelled worker congestion, a dynamic condition that causes blocking and delays. The main aim of their model is to minimize inventory, labour and transportation costs of the distribution network. For solving small-sized problems they used FICO Xpress, while they developed a heuristic method to solve the industry-sized problems with up to 500 stores and 1,000 products. They highlighted that a multi-warehouse consideration will be the next step for their research.

Gzara et al. (2014) focused on the location and allocation decisions of warehouses and inventory management under demand uncertainty on a three-level supply chain network that includes a main distribution centre, a single transport mode and multiple warehouses, customers and products. Aiming at optimizing inventory, transportation and facility establishment costs, they developed two MINLP models providing an equivalent linear formulation under service level requirements for the combination part-warehouse, and an approximate linear formulation under part service level requirements. They used CPLEX optimization software for the computational experiments. For further research, they proposed a model consisting multiple distribution centres with limited inventories, different service level measures, and part bundling or consolidation.

Ghorbani and Jokar (2016) addressed a multi-product and multi-period location-routing-inventory problem considering decisions for inventory, location-allocation and routing in a three-level supply chain including multiple suppliers, warehouses, customers and vehicles. To describe their problem, they proposed a Mixed-Integer Programming (MIP) model that minimizes operational, routing and inventory costs over all time periods. They solved their model by developing a hybrid heuristic algorithm for their large-sized examples and CLPEX for the smaller ones.

Yao et al. (2010) considered a facility location–allocation and inventory problem, in which the inventory of each SKU in each warehouse may be replenished by multiple plants. Their network includes multiple suppliers/plants, warehouses and customers. In this problem, multiple products are produced by several plants and each customer is related to a certain demand and an amount of safety stock that must be maintained to achieve certain service level, which is determined by the customer of the 3PL company. Their problem is to determine the number, the location and the inventory level of warehouses and the allocation of customer demand. This problem was formulated by a MINLP model to minimize the total inventory and transportation costs. An iterative heuristic method was developed to solve it. They applied their model and method to a chemicals company that selects among the available warehouses of a third-party logistics service provider. For future research, they proposed to consider other ways of expressing actual transportation costs (i.e., fixed and per-unit transportation cost), instead of proportional transportation cost.

Shahabi et al. (2014) examined a three-level location-inventory problem in which client demand among retailers is correlated. Their formulation includes multiple plants, warehouses and retailers, and the aim of the model is to minimize the facility location, transportation and inventory costs considering a single period. They formulated their model as a binary integer nonlinear program which was transformed to a Mixed Integer Conic Quadratic Program (MICQP). The authors proposed an outer approximation-based algorithm and demonstrated the algorithmic efficiency for this class of programs. A possible extension of this work, according to the authors, could be the incorporation of inventory control of retailers and plants.

Shahabi et al. (2013) developed mathematical models to bring together facility location and inventory control for a four-echelon single period supply chain network that comprises multiple suppliers, warehouses, hubs and customers. Firstly, they formulated the problem as an MINLP and then they reformulated it as a Conic Quadratic Mixed-Integer Program (CQMIP) to take advantage of the recent advances in solving conic programs in commercial solvers. Their objective is to minimize the facility location, transportation and the inventory costs of the four-echelon supply chain. They highlighted that capacity and congestion considerations will be possible extensions of their work.

Guo et al. (2018) examined a location-inventory-routing single period problem in a closed-loop supply chain that includes multiple suppliers, warehouses, customers and vehicles. The problem is formulated as an Integer Non-Linear Programming (INLP) model to optimize facility location, inventory control, and vehicle routing decisions for a single product and a single type of vehicles. They developed a heuristic approximation, Adaptive Hybrid Simulated Annealing Genetic Algorithm (AHSAGA) as a solution method. For further research they proposed to: (a) allow a many-to-many relationship between vehicles and retailers, relaxing the assumption that a retailer will be visited by a vehicle every working day, (b) consider secondary markets as they have become an important channel to sell used products and (c) incorporate more scenarios, such as supply risk and multiple sourcing.

Seyedhosseini et al. (2014) addressed a single-product, single-period location-routing-inventory problem considering decisions for inventory, location-allocation and routing in a three-level supply chain. The latter comprises multiple distribution centers with random disruptions, customers and vehicles. To formulate their problem, they proposed a MINLP, which minimizes facility location, transportation and inventory costs. The proposed model has been applied to a real case study of a soft drink company and is solved by a meta-heuristic methodology. The authors provided a comprehensive sensitivity analysis. For future research, they proposed to: (a) consider the frequency and the duration of random disruptions, (b) develop the model for three-level supply chains, considering product distribution to customers through warehouses or directly from suppliers to customers and (c) consider customer differentiation with partial backordering.

Diabat et al. (2017) examined a network design problem for a supply chain with a single supplier, multiple distribution centers (DCs) and multiple retailers. They formulated the problem as an MINLP model that determines the number and the location of DCs, assigns customers to DCs and indetifies the size and the timing of orders for each DC. Their objective is to minimize the supply chain cost that includes DC location cost, transportation cost between DCs and customers, purchase cost from a supplier, shortage cost, ordering cost, holding cost and lost sales cost for unserved demand. To solve the model, they used a hybrid solution algorithm based on a metaheuristics technique. To validate the model, several computational experiments were performed.

Saragih et al. (2019) developed a MINLP model and a heuristic method for the location-inventory-routing problem in a three-echelon supply chain system where location, inventory and transportation decisions are made. The network comprises a single supplier, multiple warehouses and retailers. Their single period and product model with probabilistic demand aims to minimize holding, ordering, shortage, warehouse establishment and transportation costs. The proposed heuristic was applied in a food supply chain company. The authors presented a comprehensive sensitivity analysis, showing that this company is very sensitive to the ordering cost parameter. For future research they proposed to include time windows, multiple modes, multiple products and multiple supplier considerations.

Dehghani et al. (2017) developed a mathematical model to design a three-level supply chain network, in which a single supplier sends one product from multiple distribution centres to multiple retailers with several types of vehicles. The problem is formulated as a MINLP that aims to minimize purchase, shortage, ordering, holding costs, the establishment cost of distribution centres and the transportation cost among all entities with different types of vehicles. The problem deals with major supply chain issues: (a) location-allocation, (b) vehicle routing and (c) inventory planning. Due to the computational complexity of the problem, they developed a Simulated Annealing (SA) algorithm to solve larger-scale problems. As a possible extension of this model, the authors proposed the consideration of multiple products instead of a single one.

Hiassat et al. (2017) examined a location-inventory-routing model for perishable products that identifies the number and the location of the required warehouses, the inventory level at each retailer, and the routes traveled by each vehicle. In their multi-period model, they considered multiple warehouses, customers, routes and vehicles. They created a genetic algorithm approach to solve the problem that they validated through a number of randomly generated instances were. Multiple product considerations could be an extension of their model, among others, as per the authors.

In Table 2.1 we present the recent literature on the location-inventory-routing problem classified around eight main aspects: (a) The scope of the location-inventory-routing model, (b) the solution method, (c) the breadth of the model in relation to the multiplicity of entities addressed, (d) capacity considerations, (e) cost considerations, (f) decision variables, (g) the applicability of the model and the related approach to realistic practical cases. More specifically:

- The objective function aspect in Table 2.1 deals with the scope of the model with respect to economic aspects, such as minimization of cost (MC) and maximization of profit (MP).
- The solution method aspect in Table 2.1 refers to the method that the authors used to solve their numerical problems/examples (validation tests or the case study).
- The capacity, cost and decision attributes of Table 2.1 refer to critical parameters and decision variables addressed by the design method.
- Finally, the last column of Table 2.1 is related to the applicability of the location-inventory-routing models to practical industrial cases.

According to Table 2.1 and the analysis of the literature encompassing other works, we observe the following: Regarding multiplicity, several models deal with multiple warehouses and customers, while a larger number of models deal with multiple products and suppliers. However, only a few papers deal with multiple transportation types and time periods.

In terms of capacity, the majority of the literature models that we examined include warehouse capacity consideration, fewer models deal with transportation and supplier capacities. We have not come across of any work that model's lower bound capacity for vehicles. This is not difficult to model, but it is an important practical parameter, that secures appropriate vehicle fill rates. Furthermore, very few models consider the selection of the type of vehicles.

In terms of costs, all the models we analyzed consider inventory and transportation costs. Fewer papers consider warehousing costs, that is, receiving, picking and shipping costs. Very few models dealt with the choice of transport type selection. Finally, there are few papers that included a case study.

Our work deals with all important issues a 3PL company faces in optimizing its network. Unique aspects, in addition to those dealt by various authors include the lower bound constraint in vehicle fill, and the various warehousing costs considered. The value of the work is this completeness of decisions considered, as well as the model validation through a comprehensive case study.

Table 2. 1 Key features of single objective location-inventory-routing models in recent literature

#	**Reference**	**Goal**	**Solution method**	**Multiplicity**						**Capacities**					**Costs**				**Decisions**				**Case study**
				TP	**P**	**S**	**W**	**C**	**T**	**S**	**W**	**SS**	**T**	**LB**	**Receiving**	**Picking & Shipping**	**Inventory**	**Transportation**	**Inventory quantity**	**Warehouse selection**	**Transport quantity**	**Transport type selection**	
1	Hamedani et al. 2012	MC	ECM & MOPSO		•	•	•	•		•	•					•	•	•	•	•	•		
2	Forouzanfar et al. 2012	MC	MINLP & GA	•	•	•	•	•			•	•	•				•	•	•	•	•		
3	Sainathuni et al. 2014	MC	MINLP	•	•	•		•			•		•				•	•	•		•		
4	Gzara et al. 2014	MC	MNLIP		•		•	•				•					•	•	•	•	•		
5	Ghorbani et al. 2016	MC	MIP & Heuristic	•	•	•	•	•		•	•		•		•	•	•	•	•	•	•		
6	Yao et al. 2010	MC	MINLP & Heuristic		•	•	•	•		•		•					•	•	•	•	•		•
7	Shahabi et al. 2014	MC	MINLP & OA			•	•	•			•	•				•	•	•	•	•	•		
8	Shahabi et al. 2013	MC	MINLP & CQMIP			•	•	•				•				•	•	•	•	•	•		
9	Guo et al. 2018	MC	INLP & AHSAGA			•	•	•					•		•	•	•	•	•	•	•		
10	Seyedhosseini et al. 2014	MC	MINLP & GA				•	•	•		•	•	•		•		•	•	•	•	•	•	•
11	Diabat, et al. 2017	MC	MINLP & SA				•	•			•					•	•	•	•	•	•		
12	Saragih et al. 2019	MC	MINLP & SA				•	•			•	•	•			•	•	•	•	•	•		•
13	Dehghani et al. 2017	MC	MINLP & SA				•	•	•		•	•	•			•	•	•	•	•	•	•	
14	Hiassat et al. 2017	MC	MIP & GA	•			•	•	•			•	•				•	•	•	•	•		
15	The proposed location-inventory-routing	MC	MILP	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•

Notation: MC: Minimize cost, MP: Maximize profit, MO: Multi objective, TP: Time period, P: Product, S: Supplier, W: Warehouse, C: Customer, T: Transportation types, SS: Safety stock, LB: Lower bound of transportation, ECM: Epsilon Constraint Method, MOPSO: meta-heuristic algorithm, Mixed integer conic quadratic program (MICQP), Simulated Annealing (SA), PL: Piecewise Linearization, MIP: Mixed-integer program, GA: Genetic Algorithm, OA: Outer approximation, CQMIP: Conic Quadratic Mixed-Integer Program, AHSAGA: Adaptive Hybrid Simulated Annealing Genetic Algorithm

Chapter 3. Design of the location-inventory-routing network

In this Chapter, we propose a comprehensive model for the location-inventory-routing problem that minimizes the location-inventory-routing cost from a third-Party Logistics (3PL) company perspective.

3.1 Problem description

We examine the optimization of the supply chain network from a third-Party Logistics (3PL) company perspective focusing on key location-inventory-routing aspects. To do that, we developed a novel Mixed Integer Linear Programming (MILP) model that covers the most significant decisions involved in efficient re-design of a supply chain network considering multiplicity of time periods, products, warehouses, customers, suppliers, transportation types. The model is able to support the designer to make the following decisions for each time period of the planning horizon:

(a) Which warehouse should be served by each supplier?
(b) Which warehouse will be selected to store the products and at what level (inventory)?
(c) Which warehouse should serve each customer?
(d) Which vehicle would be suitable to transport the products from the suppliers to the warehouses to the final customers?

Warehousing costs include receiving, inventory, picking and shipping costs. Transportation costs include the route cost from the suppliers to the customers through the warehouses.

More specifically, the problem addressed here concerns 3PL companies that optimize their location-inventory-routing network over a typically long planning horizon in order to satisfy the product demand. This location-inventory-routing problem considers the following entities and activities: (a) product (customer) demand, (b) suppliers, (c) warehouses, (d) transportation.

Product (customer) demand
Customers may include wholesalers, retailers, stores etc. Their locations and their demand per period of the design time horizon are considered to be known.

Suppliers
The suppliers provide products of different types. Each supplier can provide up to a maximum quantity per product and per time period.

Warehouses

The warehouses receive products from the suppliers. We consider that the warehouses are characterized by a maximum facility capacity (i.e., in pallets). The number of products carried in a unit load (i.e., pallet) varies among product pallets. In terms of inventory management, each product is related to a minimum safety stock level.

Transportation

Regarding transportation, there are no capacity considerations. Transportation demand is quantified in unit loads (i.e., pallets) of a certain product within a certain time period. Since the number of units of a product per unit load (i.e., pallet) is given, the transportation cost per transportation type between two nodes may be defined either per unit load (in pallets) or per product unit (i.e., box or piece). The transportation types we consider in the model are the following: (a) a 7-pallet van, (b) an 18-pallet truck, (c) a 33-pallet truck, (d) any combination of two trucks (e.g., 7 and 7 or 7 and 18, or …, or 33 and 33). These transportation types are indicative, and combinations of any number of trucks and types may be defined and used in general. In the case study of Chapter 4, there is no occasion in which more than two vehicles arrive from a certain supplier to a certain warehouse in the same period (day), or from a certain warehouse to a certain customer.

To solve the problem described above, we propose a novel Mixed Integer Linear Programming (MILP) model that is described in Section 3.2. In order to validate the proposed model, we have applied it to a number of cases. All tests and the related results are presented in Section 3.3.

3.2 Mathematical model

Prior to setting up the formulation, comprising the objective function and the related constraints, we first present the definition of the necessary elements of mathematical model, nodes, arcs, parameters, and decision variables.

Network

Nodes	S: *Set of existing suppliers*
	W: *Set of existing warehouses*
	C: *Set of existing customers*
	$N = S \cup W \cup C$
Arcs	$A_f = \{(i,j) \mid i \in S, j \in W\} \cup \{(i,j) \mid i \in W, j \in C\}$: *Set of transportation arcs*
Products	P: *Set of existing products*
Time periods	T: *Set of time periods*

Parameters

It concerns the following sets: (a) costs; receiving, picking and shipping, inventory, transportation, (b) capacities; warehouse, supplier, vehicle, products, (c) demand of products in pallets and (d) the safety stock levels for each product and the inventory of the initial period.

Costs	
Receiving pallet cost	R_w: *Pallet receiving and placement cost in warehouse* $w \in W$
Picking & Shipping cost	S_w: *Pallet picking and shipping cost in warehouse* $w \in W$
Inventory cost	N_{pw}: *Daily inventory cost of (a pallet of) product* $p \in P$ *in warehouse* $w \in W$
Transportation cost	E_{ij}^{v}: *Cost of a trip made by vehicle type combination* $v \in V$ *from node* i *to node* j *where* $(i,j) \in A$

Capacities	
Warehouse capacity	Q_w: *Capacity of warehouse* $w \in W$ *(in pallets)*
Supplier capacity	Q_{spt}: *Capacity of supplier* $s \in S$ *of product* $p \in P$ *at period* $t \in T$
Transportation type capacity	M^v: *Total capacity of the vehicle type combination* $v \in V$
Lower bound	F^v: *Lower bound of loading factor of vehicle type combination* $v \in V$ *(may be zero for the smallest vehicle type)*

Products & Demand	
Products	$P^s \subseteq P$: *Products provided by supplier* $s \in S$
Demand	D_{ptc}: *Pallet demand of product* $p \in P$ *in* $t \in T$ *for customer* $c \in C$

Inventory parameters	
Safety stock	L_p: *Safety stock of product* $p \in P$ *(in pallets). Note this indicates stock across all warehouses*
Initial inventory	i_{p0w}: *Initial inventory of product* $p \in P$*, in warehouse* $w \in W$ *at time* $t = 0$

Transportation types	
	V: *Set of transportation types i.e., available vehicles and possible vehicle combinations i.e.,*
	v *= 1 is a 7-pallet van* v *= 2 is an 18-pallet truck* v *= 3 is a 33-pallet truck*

	v = 4 is the combination of 7-pallet van and 7-pallet van
	v = 5 is the combination of 7-pallet van and 18-pallet truck
	v = 6 is the combination of 7-pallet van and 33-pallet truck
	v = 7 is the combination of 18-pallet truck and 18-pallet truck
	$v = 8$ is the combination of 33-pallet truck and 18-pallet truck
	$v = 9$ is the combination of two 33-pallet trucks
	$v = 10$ is the combination of three 33-pallet trucks
	$v = 11$ is the combination of four 33-pallet trucks
	$v = 12$ is the combination of five 33-pallet trucks
	$v = 13$ is the combination of six 33-pallet trucks
	$v = 14$ is the combination of seven 33-pallet trucks
	$v = 15$ is the combination of eight 33-pallet trucks
	$v = 16$ is the combination of nine 33-pallet trucks
	$v = 17$ is the combination of ten 33-pallet trucks
	$v = 18$ is the combination of eleven 33-pallet trucks
	$v = 19$ is the combination of twelve 33-pallet trucks
	$v = 20$ is the combination of thirteen 33-pallet trucks

Decision variables

Quantities transported	*x_{ptij}^{v}: Number of pallets of product $p \in P$ transported by vehicle type combination $v \in V$ in period (i.e., day) $t \in T$ from node i to node j where $(i, j) \in A_f$*
Inventory	*i_{ptw}: Inventory (in pallets) of product $p \in P$ at the end of day $t \in T$ in warehouse $w \in W$*
Vehicle Usage	*m_{tij}^{v}: Takes value 1, if vehicle type combination $v \in V$ is used at time t to traverse arc $(i, j) \in A$, otherwise it takes value 0*

The mathematical formulation of the proposed location-inventory-routing model is presented below.

Objective function (3.1) minimizes total transportation and warehousing costs (inventory and processing costs) over the entire time period. Transportation costs include both the costs of transporting the goods from the suppliers to the warehouses and the corresponding costs from the warehouses to the final customers. Total cost comprises the following parts:

(a) Inventory cost: ($\sum_{t \in T} \sum_{p \in P} \sum_{w \in W} N_{pw} i_{pwt}$),

(b) Processing cost: ($\sum_{t \in T} \sum_{p \in P} \sum_{s \in S} \sum_{w \in W} \sum_{v \in V} R_w x_{ptsw}^{v} + \sum_{t \in T} \sum_{p \in P} \sum_{w \in W} \sum_{c \in C} \sum_{v \in V} S_w x_{ptwc}^{v}$),

(c) Transport cost: ($\sum_{t\in T}\sum_{v\in V}\sum_{(i,j)\in A} E_{ij}^{v} m_{tij}^{v}$).

$$min\sum_{t\in T}\sum_{p\in P}\sum_{w\in W} N_{pw} i_{pwt} + \sum_{t\in T}\sum_{p\in P}\sum_{s\in S}\sum_{w\in W}\sum_{v\in V} R_w x_{ptsw}^{v} + \sum_{t\in T}\sum_{p\in P}\sum_{w\in W}\sum_{c\in C}\sum_{v\in V} S_w x_{ptwc}^{v} + \sum_{t\in T}\sum_{v\in V}\sum_{(i,j)\in A} E_{ij}^{v} m_{tij}^{v} \quad (3.1)$$

The constraints of the proposed model are presented in groups as follows:

Customer demand

Constraint (3.2) ensures that the demand D_{ptc} of customer $c \in C$ for each product $p \in P$ is satisfied with produced quantities x_{ptwc}^{v} during each time period $t \in T$.

$$\sum_{w\in W}\sum_{v\in V} x_{ptwc}^{v} \geq D_{ptc}, \quad p \in P, c \in C, t \in T \quad (3.2)$$

Transportation routes

Constraint (3.3) ensures that the transported product quantities x_{ptwc}^{v} along arc $(i,j) \in A_f$, must be greater than or equal to the lower bound set by the loading factor F^v of the selected vehicle type combination $v \in V$. Constraint (3.4) ensures that the transported product quantities x_{ptij}^{v} must be less or equal to the total capacity M^v of the vehicle type combination $v \in V$, in period $t \in T$.

$$\sum_{p\in P} x_{ptij}^{v} \geq F^v M^v m_{tij}^{v}, \quad (i,j) \in A_f, t \in T, v \in V \quad (3.3)$$

$$\sum_{p\in P} x_{ptij}^{v} \leq M^v m_{tij}^{v}, \quad (i,j) \in A_f, t \in T, v \in V \quad (3.4)$$

Inventory

Constraint (3.5) defines the evolution of inventory i_{ptw} of warehouse $w \in W$: that is, i_{ptw} is equal to the inventory $i_{p(t-1)w}$ of the previous period plus the quantities received from the suppliers x_{ptsw}^{v} minus the ones shipped to the customers x_{ptwc}^{v} at time period $t \in T$. Constraint (3.6) ensures that the total inventory of product $p \in P$ in all warehouses $w \in W$, is greater or equal to the safety stock L_p that is set for this product $p \in P$ at period $t \in T$.

$$i_{ptw} = i_{p(t-1)w} + \sum_{s\in S}\sum_{v\in V} x_{ptsw}^{v} - \sum_{c\in C}\sum_{v\in V} x_{ptwc}^{v}, \quad p \in P, w \in W, t \in T \quad (3.5)$$

$$\sum_{w \in W} i_{ptw} \geq L_p, \quad p \in P, t \in T \tag{3.6}$$

Warehouse capacity

Constraint (3.7) ensures that the total inventory of all products in warehouse $w \in W$ is less than or equal to the total capacity $Q_w$of this warehouse (in pallets) at time period $t \in T$.

$$\sum_{p \in P} i_{ptw} \leq Q_w, \quad \in W, t \in T \tag{3.7}$$

Supplier capacity

Constraint (3.8) ensures that the total quantities x_{ptij}^v of product $p \in P$ supplied by supplier $s \in S$ must be less or equal to the capacity Q_{spt} of supplier $s \in S$ (in pallets) of product $p \in P$ in period $t \in T$.

$$\sum_{v \in V} \sum_{w \in W} x_{ptij}^v \leq Q_{spt,} \quad p \in P, t \in T, s \in S \tag{3.8}$$

Variables

Constraint (3.9) defines the vehicle usage m_{tij}^v variable to be binary, receiving the value 1, if vehicle type combination $v \in V$ is used along arc $(i,j) \in A$ in period $p \in P$ and 0 otherwise. Constraint (3.10) ensures that product quantities x_{ptij}^v shipped by vehicle combination $v \in V$ along arc $(i,j) \in A_f$, in time period $t \in T$, must be greater or equal to 0. Constraints (3.11) ensure that the inventory variables i_{ptw} of product $p \in P$ at the end of time period $t \in T$ in warehouse $w \in W$ must be greater or equal to 0.

$$m_{tij}^v \in \{0,1\}, \qquad v \in V, (i,j) \in A \tag{3.9}$$

$$x_{ptij}^v \geq 0, \quad v \in V, t \in T, (i,j) \in A_f, \tag{3.10}$$

$$i_{ptw} \geq 0, \qquad p \in P, t \in T, w \in W \tag{3.11}$$

3.3 Model validation

The model was implemented in Python PuLP, using a PC equipped with a 3.40 GHz Intel Core i7 and 8.0 GB of RAM. It was solved by the commercial MILP solver Gurobi Optimizer version 9.1.2 (Gurobi Optimization, 2021).

A series of tests have been performed in order to validate the consistency and correctness of the proposed model. These validation tests considered various simple cases comprising two suppliers, two warehouses, two customers, two products and three vehicles with various capacities.

The validation cases are presented in Table 3.1 and described in detail below. The table also shows per case, which part of the model was validated and the model attributes tested.

Case (a) - validation of supplier capacity and safety stock constraints

In this case two time periods (T=2) were considered.

- Supplier 1 may not supply product 2 ($Q_{12t} = 0$) and supplier 2 may not supply product ($Q_{21t} = 0$).
- The demand of customers 1 and 2 is 21 pallets of product 1 per period in both time periods ($D_{111} = 21,\ D_{121} = 21, D_{112} = 21, D_{122} = 21$). The demand for product 2 is zero for customer 1 in both time periods and 7 pallets per period for customer 2 in both time periods ($D_{211} = 0,\ D_{221} = 0,\ D_{212} = 7, D_{222} = 7$).
- The safety stock of products 1 and 2 is 2007 and 2014 pallets ($L_1 = 2007, L_2 = 2014$), respectively, and is higher than the inventory of those products on day 0 (2000 pallets for product 1 and 2000 pallets for product 2) ($i_{10w} = 2000, i_{20w} = 2000$). Therefore, appropriate quantities of products must be transported from the suppliers, to satisfy the demand and safety stock constraints.
- The capacities of the vehicles are 7 pallets for vehicle 1, 14 for vehicle 2 and 21 for vehicle 3 ($M^1 = 7, M^2 = 14, M^3 = 21$).

We confirmed the expected solution:

- Supplier 1 ships only pallets of product 1 and supplier 2 ships only pallets of product 2.
- The quantities that are transported from suppliers to the warehouses are 119 pallets ($x^v_{ptsw} = 119$) and the customer demand is 98 pallets ($D_{ptc} = 98$). The difference of 21 pallets is equal to the safety stock deficit of the products.

Case (b) - validation of the vehicle loading factor constraint

Two time periods (T=2) were considered.

- Supplier 1 may not supply product 2 ($Q_{12t} = 0$) and supplier 2 may not supply product 1 ($Q_{21t} = 0$).

- The demand of customer 1 is 13 pallets of product 1 per period in both time periods. The demand of customer 2 for product 1 is zero ($D_{111} = 13,\ D_{121} = 13, D_{112} = 0, D_{122} = 0$). The demand for product 2 is zero for customer 1 in both time periods and 7 pallets per period for customer 2 in both time periods ($D_{211} = 0,\ D_{221} = 0,\ D_{212} = 7, D_{222} = 7$).
- The safety stock of products 1 and 2 is 2007 and 2014 pallets ($L_1 = 2007,\ L_2 = 2014$), respectively, and is higher than the inventory of those products on day 0 (2000 pallets for product 1 and 2000 products must be transported from the suppliers, to satisfy the demand and safety stock constraints pallets for product 2) ($i_{10w} = 2000,\ i_{20w} = 2000$). Therefore, appropriate quantities of products must be transported from the suppliers, to satisfy the demand and safety stock constraints.
- The capacities of the vehicles are 7 pallets for vehicle 1, 14 for vehicle 2 and 21 for vehicle 3 ($M^1 = 7,\ M^2 = 14,\ M^3 = 21$).
- The vehicle lower bound for vehicle 2 is set to 100% of its capacity ($F^2 = 100\%$ and for the vehicle 1 and 3 is 0% ($F^1 = 0\%,\ F^3 = 0\%$).
- The warehousing costs of warehouse 1 were higher than those pf warehouse 2. Receiving and shipping costs in warehouse 1 are 1000€ ($R_1 = 1000,\ S_1 = 1000$), and those costs in warehouse 2 are 1€ ($R_2 = 1,\ S_2 = 1$).

We confirmed the expected solution:

- Shipments and receipts are made from the most cost-effective warehouse, which is warehouse 2.
- Supplier 1 ships only pallets of product 1 and supplier 2 ships only pallets of product 2.
- Vehicle 2 carried 14 pallets, more than the demand which is 13, to satisfy the loading factor constraint.

Case (c) - validation of inventory and warehouse capacity constraints

Two time periods (T=2) were considered.

- Supplier 1 may not supply product 2 ($Q_{12t} = 0$) and supplier 2 may not supply product 1 ($Q_{21t} = 0$).
- The demand of customers 1 and 2 is 21 pallets of product 1 per period in both time periods ($D_{111} = 21,\ D_{121} = 21,\ D_{112} = 21,\ D_{122} = 21$). The demand for product 2 is zero for customer 1 in both time periods and 7 pallets per period for customer 2 in both time periods ($D_{211} = 0,\ D_{221} = 0,\ D_{212} = 7,\ D_{222} = 7$).
- The safety stock of products 1 and 2 is 1007 and 1014 pallets ($L_1 = 1007,\ L_2 = 1014$), respectively, and is higher than the inventory of those products on day 0 (1000 pallets for

product 1 and 1000 pallets for product 2 must be transported from the suppliers, to satisfy the demand and safety stock ($i_{10w} = 1000,\ i_{20w} = 1000$). Therefore, appropriate quantities of products must be transported from the suppliers, to satisfy the demand and safety stock constraints.

- The capacities of the vehicles are 7 pallets for vehicle 1, 14 for vehicle 2 and 21 for vehicle 3 ($M^1 = 7,\ M^2 = 14,\ M^3 = 21$).
- The warehousing costs are the same in both warehouses. Receiving and shipping costs in warehouses 1 and 2 are 1€ ($R_1 = 1,\ S_1 = 1, = 1,\ S_2 = 1$).
- Warehouse 1 capacity is set on 8000 pallets ($Q_1 = 8000$) and its initial inventory is 2000 ($i_{p01} = 2000$) for both products. Warehouse 2 has not capacity ($Q_2 = 0$, so in effect is not operational) and of course no inventory ($i_{p02} = 0$).

We confirmed the expected solution:

- Supplier 1 ships only pallets of product 1 and supplier 2 ships only pallets of product 2.
- Warehouse 2 can not used to store pallets.
- The quantities that are transported from suppliers to warehouse 1 are 119 pallets ($x^v_{pts1} = 119$) and the customer demand is 98 pallets ($D_{ptc} = 98$). The difference of 21 pallets is equal to the safety stock deficit of the products.

Case (d) - validation of vehicle capacity constraint

Two time periods (T=2) were considered.

- Supplier 1 may not supply product 2 (zero capacity) ($Q_{12t} = 0$) and supplier 2 may not supply product 1 ($Q_{21t} = 0$).
- The demand of customers 1 and 2 is 21 pallets of product 1 per period in both time periods ($D_{111} = 21,\ D_{121} = 21,\ D_{112} = 21,\ D_{122} = 21$). The demand for product 2 is zero for customer 1 in both time periods and 7 pallets per period for customer 2 in both time periods ($D_{211} = 0,\ D_{221} = 0,\ D_{212} = 7,\ D_{222} = 7$).
- The safety stock of products 1 and 2 is 2007 and 2014 pallets ($L_1 = 2007,\ L_2 = 2014$), respectively, and is higher than the inventory of those products on day 0 (2000 pallets for product 1 and 2000 products must be transported from the suppliers, to satisfy the demand and safety stock constraints for product 2) ($i_{10w} = 2000,\ i_{20w} = 2000$).
- The capacities of the vehicles are 7 pallets for vehicle 1, 14 for vehicle 2 and 21 for vehicle 3 ($M^1 = 7,\ M^2 = 14,\ M^3 = 21$).
- Receiving and shipping costs in warehouse 1 are 2€ ($R_1 = 2,\ S_1 = 2$), and those costs in warehouse 2 are 1€ ($R_2 = 1,\ S_2 = 1$).

We confirmed the expected solution:

- Supplier 1 ships only pallets of product 1 and supplier 2 ships only pallets of product 2.
- All products are transferred with vehicle 3, which has the appropriate capacity.
- Shipment and receipt operations are performed from the most cost-effective warehouse, that is warehouse 2.
- The quantities that are transported from suppliers to warehouse 1 are 119 pallets ($x^{v}_{pts1} = 119$) and the customer demand is 98 pallets ($D_{ptc} = 98$). The difference of 21 pallets is equal to the safety stock deficit of the products.

Case (e) - validation of customer demand and transporting route constraints

Two time periods (T=2) were considered.

- Supplier 1 may not supply product 2 ($Q_{12t} = 0$) and supplier 2 may not supply product 1 ($Q_{21t} = 0$).
- The demand of customers 1 and 2 is 21 pallets of product 1 per period in both time periods ($D_{111} = 21,\ D_{121} = 21,\ D_{112} = 21,\ D_{122} = 21$). The demand for product 2 is zero for customer 1 in both time periods and 7 pallets per period for customer 2 in both time periods ($D_{211} = 0,\ D_{221} = 0,\ D_{212} = 7,\ D_{222} = 7$).
- The safety stock of products 1 and 2 is 2007 and 2014 pallets ($L_1 = 2007,\ L_2 = 2014$), respectively, and is higher than the inventory of those products on day 0 (2000 pallets for product 1 and 2000 products must be transported from the suppliers, to satisfy the demand and safety stock constraints for product 2) ($i_{10w} = 2000, i_{20w} = 2000$).
- Vehicle capacities are 7 pallets for vehicle 1, 14 for vehicle 2 and 21 for vehicle 3 ($M^1 = 7,\ M^2 = 14,\ M^3 = 21$). The minimum loading factor is zero in all three vehicles ($F^1 = 0\%,\ F^2 = 0\%,\ F^3 = 0\%$).
- The warehousing costs are the same for both warehouses. Receiving and shipping costs in warehouse 1 are 1€ ($R_1 = 1,\ S_1 = 1$), and those costs in warehouse 2 are also 1€ ($R_2 = 1,\ S_2 = 1$).
- The transportation costs per vehicle from warehouses to customers have large differences. The costs from warehouse 1 to customer 1 are as follows: for vehicle 1: $E^1_{11} = 900€$, for vehicle 2: $E^2_{11} = 1500€$, for vehicle 3: $E^3_{11} = 2100€$. From warehouse 1 to customer 2 the costs are: for vehicle 1: $E^1_{12} = 920€$, for vehicle 2: $E^2_{12} = 1540€$, for vehicle 3: $E^3_{12} = 2180€$. Similarly, $E^1_{21} = 91€$, $E^2_{21} = 152€$ and $E^3_{21} = 214€$, $E^1_{22} = 93€$,$E^2_{22} = 156€$, $E^3_{22} = 222€$).

We confirmed the expected solution:

- Supplier 1 ships only pallets of product 1 and supplier 2 ships only pallets of product 2.
- Shipments towards the customers are all made from warehouse 2, due to the high distribution costs from warehouse 1.
- The quantities that are transported from suppliers to warehouse 1 are 119 pallets ($x^{v}_{pts1} = 119$) and the customer demand is 98 pallets ($D_{ptc} = 98$). The difference of 21 pallets is equal to the safety stock deficit of the products.

Case (f) - validation of customer demand and supplier capacity constraints

Two time periods (T=2) were considered.

- Supplier 1 may not supply product 2 ($Q_{12t} = 0$) and supplier 2 may not supply product 1 ($Q_{21t} = 0$).
- The demand of customers 1 and 2 is 21 pallets of product 1 per period in both time periods ($D_{111} = 21,\ D_{121} = 21,\ D_{112} = 21,\ D_{122} = 21$). The demand for product 2 is zero for customer 1 in both time periods and 7 pallets per period for customer 2 in both time periods ($D_{211} = 0,\ D_{221} = 0,\ D_{212} = 7,\ D_{222} = 7$).
- The safety stock of products 1 and 2 is 2007 and 2014 pallets ($L_1 = 2007,\ L_2 = 2014$), respectively, and is higher than the inventory of those products on day 0 (2000 pallets for product 1 and 2000 products must be transported from the suppliers, to satisfy the demand and safety stock constraints for product 2) ($i_{10w} = 2000, i_{20w} = 2000$).
- Vehicle capacities are 7 pallets for vehicle 1, 14 for vehicle 2 and 21 for vehicle 3 ($M^1 = 7,\ M^2 = 14,\ M^3 = 21$). The minimum loading factor is zero in all three vehicles ($F^1 = 0\%,\ F^2 = 0\%,\ F^3 = 0\%$).
- The transportation costs per vehicle from warehouses to customers have large differences. The costs from warehouse 1 to customer 1 are as follows: for vehicle 1: $E^1_{11} = 70€$, for vehicle 2: $E^2_{11} = 120€$, for vehicle 3: $E^3_{11} = 180€$. From warehouse 1 to customer 2 the costs are: for vehicle 1: $E^1_{12} = 71€$, for vehicle 2: $E^2_{12} = 122€$, for vehicle 3: $E^3_{12} = 184€$. Similarly, $E^1_{21} = 720€$, $E^2_{21} = 1240€$ and $E^3_{21} = 1880€$, $E^1_{22} = 730€, E^2_{22} = 1260€$, $E^3_{22} = 1920€$).
- The warehousing costs are the same for both warehouses. Receiving and shipping costs in warehouse 1 are 1€ ($R_1 = 1,\ S_1 = 1$), and those costs in warehouse 2 are also 1€ ($R_2 = 1,\ S_2 = 1$).

We confirmed the expected solution:

- Supplier 1 ships only pallets of product 1 and supplier 2 ships only pallets of product 2.

- Shipments towards the customers are all made from warehouse 2, due to the high distribution costs from warehouse 1.
- The quantities that are transported from suppliers to warehouse 1 are 119 pallets ($x_{pts1}^{v} = 119$) and the customer demand is 98 pallets ($D_{ptc} = 98$). The difference of 21 pallets is equal to the safety stock deficit of the products.

Case (g) - validation of customer demand and safety stock demand constraints

Two time periods (T=2) were considered.

- Supplier 1 may not supply product 2 ($Q_{12t} = 0$) and supplier 2 may not supply product 1 ($Q_{21t} = 0$).
- The demand of customers 1 and 2 is 21 pallets of product 1 per period in both time periods ($D_{111} = 21,\ D_{121} = 21,\ D_{112} = 21,\ D_{122} = 21$). The demand for product 2 is zero for customer 1 in both time periods and 7 pallets per period for customer 2 in both time periods ($D_{211} = 0,\ D_{221} = 0,\ D_{212} = 7,\ D_{222} = 7$).
- The safety stock of products 1 and 2 is 2007 and 2014 pallets ($L_1 = 2007,\ L_2 = 2014$), respectively, and is higher than the inventory of those products on day 0 (2000 pallets for product 1 and 2000 products must be transported from the suppliers, to satisfy the demand and safety stock constraints for product 2) ($i_{10w} = 2000, i_{20w} = 2000$).
- Vehicle capacities are 7 pallets for vehicle 1, 14 for vehicle 2 and 21 for vehicle 3 ($M^1 = 7,\ M^2 = 14,\ M^3 = 21$). The minimum loading factor is zero in all three vehicles ($F^1 = 0\%,\ F^2 = 0\%,\ F^3 = 0\%$).
- The warehousing costs are the same for both warehouses. Receiving and shipping costs in warehouse 1 are 1€ ($R_1 = 1,\ S_1 = 1$), and those costs in warehouse 2 are also 1€ ($R_2 = 1,\ S_2 = 1$).
- The transportation costs per vehicle from warehouses to customers have large differences. The costs from warehouse 1 to customer 1 are as follows: for vehicle 1: $E_{11}^{1} = 700€$, for vehicle 2: $E_{11}^{2} = 1200€$, for vehicle 3: $E_{11}^{3} = 1800€$. From warehouse 1 to customer 2 the costs are: for vehicle 1: $E_{12}^{1} = 71€$, for vehicle 2: $E_{12}^{2} = 122€$, for vehicle 3: $E_{12}^{3} = 184€$. Similarly, $E_{21}^{1} = 720€$, $E_{21}^{2} = 1240€$ and $E_{21}^{3} = 1880€$, $E_{22}^{1} = 73€, E_{22}^{2} = 126€$, $E_{22}^{3} = 192€$).

We confirmed the expected solution:

- Supplier 1 ships only pallets of product 1 and supplier 2 ships only pallets of product 2.
- Shipments towards the customers are all made from warehouse 2, due to the high distribution costs from warehouse 1.

- The quantities that are transported from suppliers to warehouse 1 are 119 pallets ($x^{v}_{pts1} = 119$) and the customer demand is 98 pallets ($D_{ptc} = 98$). The difference of 21 pallets is equal to the safety stock deficit of the products.

Table 3. 1. Validation cases and the model attributes tested

#	Cases	Constraints Validated								Parameters							Validation scope
		CD	TR	IE	WC	SC	VLB	VC	SSD	S	W	VT	RC	IC	PSC	TC	
1	Case (a)					•			•	•							Supplier capacity and safety stock constraints
2	Case (b)						•					•	•	•	•		Vehicle lording factor
3	Case (c)			•	•								•	•	•		Inventory and warehousing constraints
4	Case (d)							•			•	•					Vehicle capacities and types
5	Case (e)	•	•								•		•		•	•	Which warehouse will ship the demand, due to costs
6	Case (f)	•				•				•					•	•	Customer demand and supplier capacity constraints
7	Case (g)	•							•						•	•	Customer demand safety stock demand constraints

CD: Customer Demand, TR: Transportation Route, IE: Inventory Evolution WC: Warehouses Capacity, SC: Suppliers Capacity, VLB: Vehicle Lower Bound, VC: Vehicles Capacity, SSD Safety Stock Demand, S: Suppliers, W: Warehouses, C: Customers, VT: Vehicle Types, C: Costs, RC: Receiving Costs, IC: Inventory Costs, PSC: Picking & Shipping Costs, TC: Transportation Costs

Chapter 4. Case study: Improvement of a 3PL network

We have applied the proposed location-inventory-routing model to an industrial case in order to investigate the model's applicability and effectiveness. More specifically, we have examined the re-design of part of the supply chain network of a large-scale Greek 3PL company. To apply the model to this case, we have used information provided directly from the company, its geographical footprint including the warehouses, its suppliers and customers, the related capacities and the products managed by the company. Any information not available from company sources was retrieved from relevant literature or was based on relevant assumptions.

Firstly, we validated all information, data and assumptions by modelling the current (as-is) supply chain network of the 3PL company and compared the company's current status and costs to the costs obtained from the proposed MILP model. During this step, some decision variables of the model were assigned fixed values to reflect the current situation.

Subsequently, we tested the ability and the efficiency of the model to re-design high performing networks by comparing the current (as-is) network with the transformed (to-be) network that emerged as the solution of the proposed location-inventory-routing model. Despite the high complexity of this case, the selected commercial solver provides optimal solutions within reasonable computational times.

4.1 Company Description

Company general characteristics

The selected company is a pioneering Greek 3PL provider that offers a full spectrum of supply chain services, including dry and cold cargo services. Also, it manages all types of consumer goods, from very smalls to heavy and bulky items. It also specializes in bonded warehouses, customs clearance services, and all types of customs procedures, as well as specialist customs consultancy services. The industries served include food, beverage, consumer goods, retail, e-commerce, technology, motor and pharmaceuticals-cosmetics.

The company has three bases located in Attica (@ Mandra, Magoula and Aspropyrgos), Thessaloniki (Sindos area where this case study has focused upon) and Achaia (Patras). It owns 30 warehouses throughout Greece, in Alexandroupolis, Kavala, Thessaloniki, Florina, Ioannina, Larissa, Attica, Achaia, Sparta, Rhodes and Heraklion Crete a total area over 130,000 m^2. Most warehouses are equipped with modern back-to-back racks, electro-hydraulic ramps with blow tubes and state-of-the-art electric forklift trucks. The company also uses total of 600 trucks provided by cooperating transport providers. The number of staff employed is 500. Regarding the financial data of the company from the annual financial statements for ear 2020, revenues amounted to 21.7 million euros and EBITDA was 2.8 million euros.

Network in Northern Greece

In this thesis we study the network in Northern Greece, which comprises: (a) three warehouses in the industrial area of Thessaloniki, (b) 23 suppliers throughout Greece that may supply one or more products to the company warehouses, and (c) 53 customers with the majority of them located in Northern Greece. The structure of the 3PL network is illustrated in Figure 4.1.

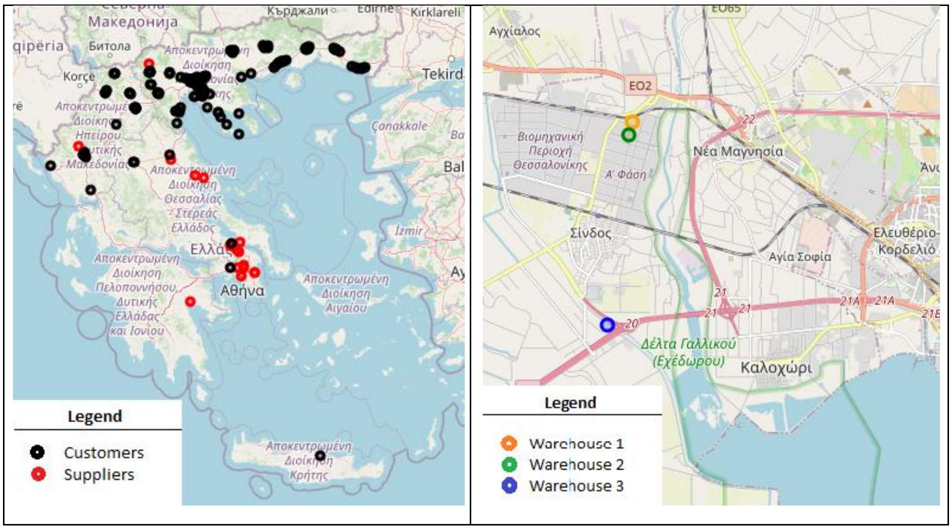

Figure 4. 1. Structure of the 3PL network under study

Products and demand of N. Greece operations

The 3PL company receives 41 products from its suppliers. All products are of the dry type (i.e., not temperature controlled nor frozen) and were classified into 13 product types. Each product type includes products with very similar product characteristics in terms of weight, size, usage. Each product is characterized by a certain pallet loading (boxes per pallet) and thus known receiving, picking, shipping, inventory and transportation costs. For example, consider a product type that comprises washing powder and laundry softener products. One pallet of this product consists of 25 boxes of laundry products. The exact costs are presented in Appendix B. The actual customer demand in number of pallets for each customer, for each day, for each product have been provided by the company.

Warehousing (and inventory) of N. Greece operations

The capacities of the three existing warehouses in N. Greece are 22,000, 7,000 and 3,000 pallets, respectively. The two higher capacity warehouses use back-to-back pallet racking while the lower capacity warehouse has no shelves; pallets are stored on the floor in blocks. Electric pallet trucks are used for material handling in all warehouses. We have assumed that the minimum stock level for each product equals to 15% of its daily demand as a realistic assumption for this market. Based on the actual inventory data at product level provided by the 3PL company, the initial inventory for all products are 5,763 pallets in Warehouse 1, 3,879 pallets in Warehouse 2 and 577 pallets in Warehouse 3. These products are stored in the available warehouses according to the requirements of each supplier and space availability. The initial inventory per product is given in Table 4.1. Warehousing costs were defined per pallet based on realistic assumptions and are presented in Appendix B.

Table 4. 1. Inventory for each product per warehouse (in pallets)

	Warehouse 1	Warehouse 2	Warehouse 3
Product 1	217		
Product 2	195		
Product 3	150		
Product 4	183		
Product 5	140		
Product 6	138		
Product 7	112		
Product 8	66		
Product 9	217		
Product 10	164		
Product 11	264		
Product 12	95		
Product 13	87		
Product 14	64		
Product 15	291		
Product 16	99		
Product 17	70		
Product 18	158		
Product 19	175		
Product 20	156		
Product 21	185		
Product 22	101		

Product 23	161		
Product 24	56		
Product 25	45		
Product 26	40		
Product 27	34		
Product 28	30		
Product 29	26		
Product 30	51		
Product 31	61		
Product 32	35		
Product 33	36		
Product 34	61		
Product 35	172		
Product 36	97		
Product 37	74		
Product 38	626	212	
Product 39	756		577
Product 40	75		
Product 41		3667	

Transportation and fleet

The suppliers are responsible for transportation between their facilities and the 3PL warehouses, while the 3PL company is responsible for transportation between warehouses and customers. The available fleet of vehicles consists of three types of trucks: 7-pallet vans, 18-pallet trucks, 33-pallet trucks. We assumed that there is no lower bound for the 7-pallet van while the 18-pallet and the 33-pallet trucks should be full at least by 70% of their maximum capacity to be available for usage.

4.2 Analysing the as-is network

We first analysed the current (as-is) network. This analysis has been used to estimate the current costs and then to compare these costs and network to the proposed one (to-be). To perform this analysis, we solved the MILP model of (3.1) to (3.11) under appropriate conditions as described below.

Inputs of the network

The inputs of the network include: the suppliers and their products, the warehouses and their capacities, the connections between suppliers, warehouses and customers, the inventory per product per warehouse in day 0, the customers and their demand, the vehicles with their capacities, the warehousing and transportation costs.

Regarding the as-is scenario, Figures 4.2-4.6 show the transportation connections among the main entities of the 3PL company (i.e., suppliers, warehouses and customers), in each day of the time horizon (i.e., 5 days of a week) respectively. More specifically, the left-hand side of those Figures presents the connections at country level while the right-hand side focuses on the wider area of Thessaloniki in North Greece. Each node represents an entity and each arc represents the connection between a supplier and a warehouse or a customer and a warehouse. More specifically, blue, green and purple nodes represent suppliers that are connected to warehouse 1, warehouse 2 and warehouse 3, respectively. Orange, red and black nodes represent customers that are connected to warehouse 1, warehouse 2 and warehouse 3, respectively.

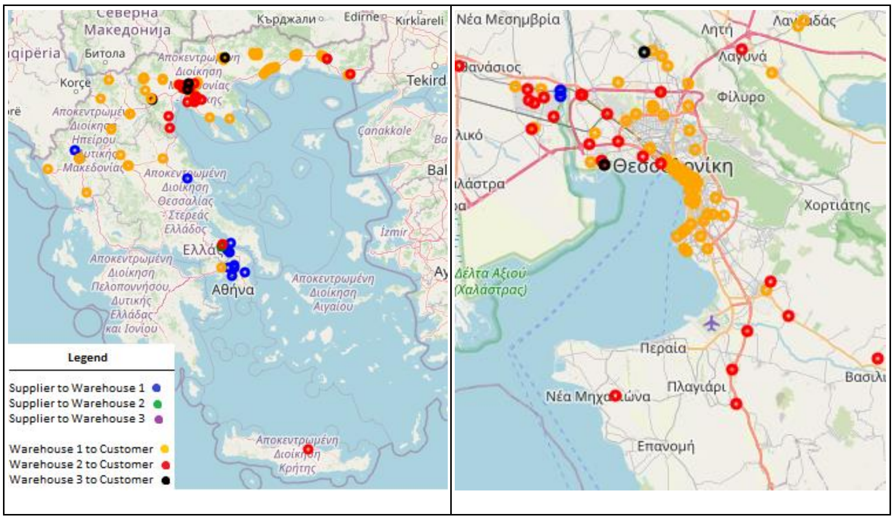

Figure 4. 2. Illustration of the as-is 3PL network in Day 1

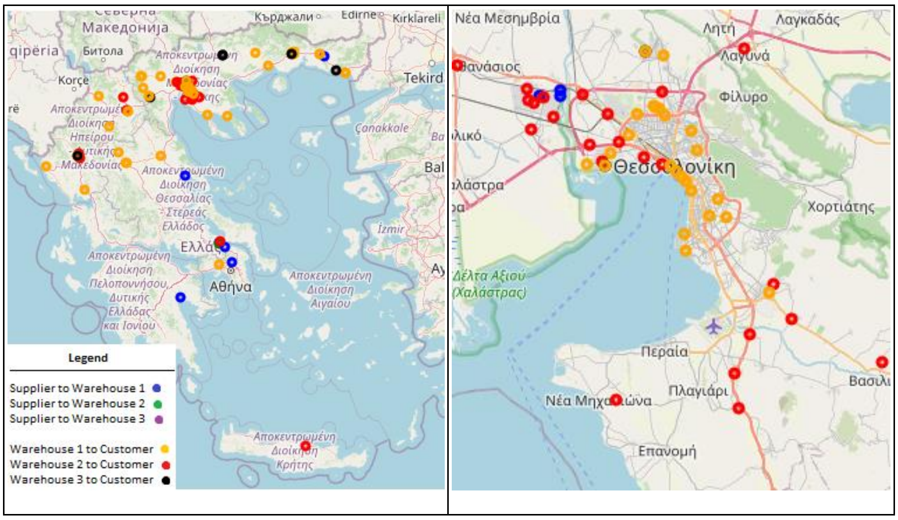

Figure 4. 3. Illustration of the as-is 3PL network in Day 2

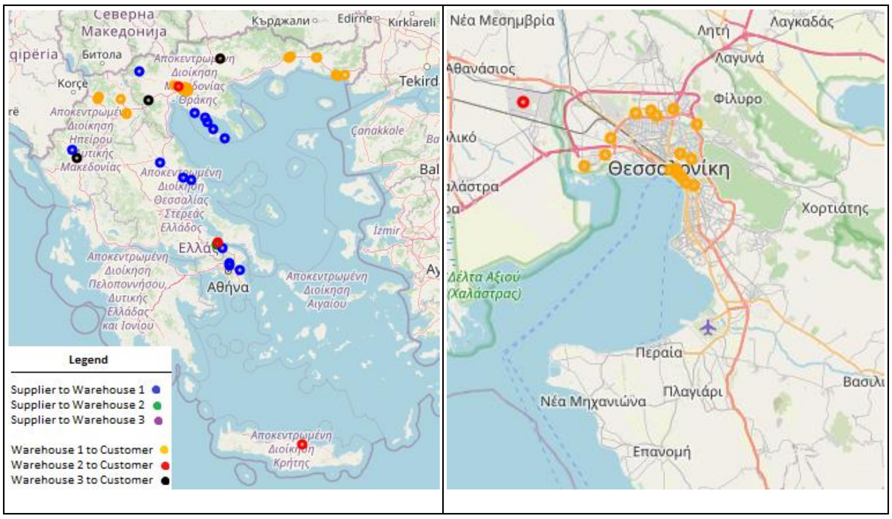

Figure 4. 4. Illustration of the as-is 3PL network in Day 3

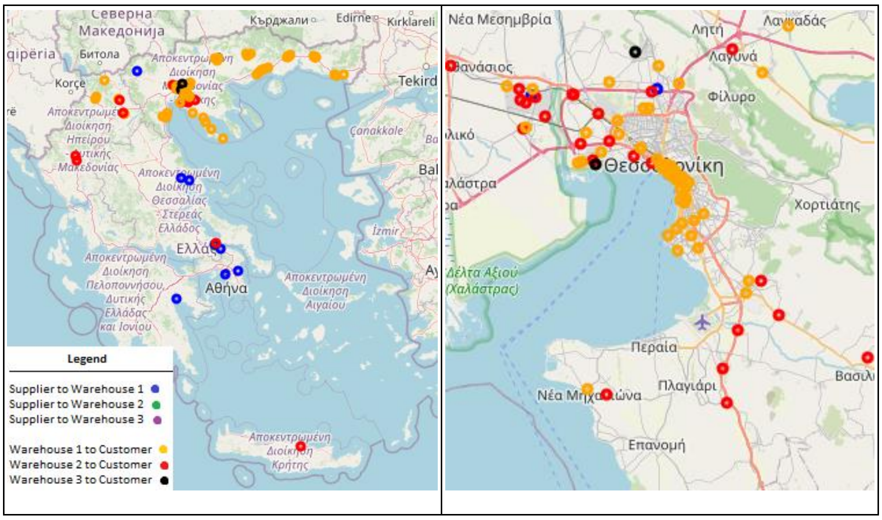

Figure 4. 5. Illustration of the as-is 3PL network in Day 4

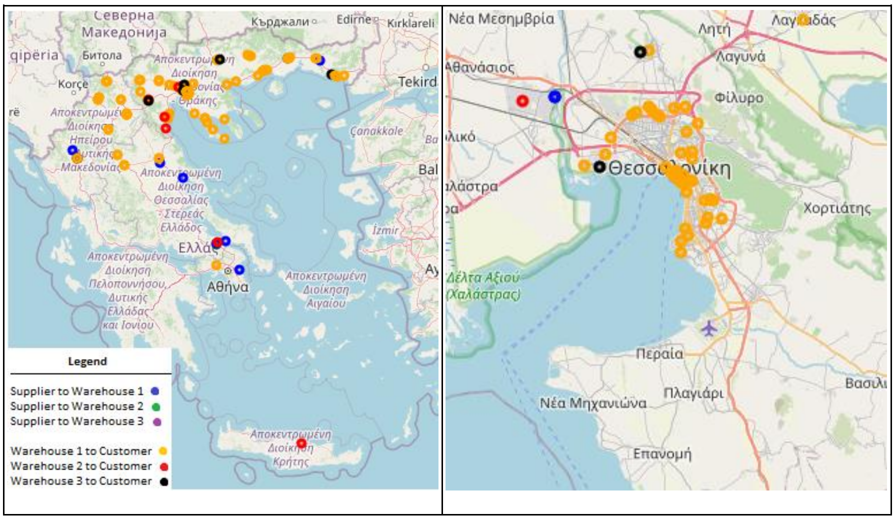

Figure 4. 6. Illustration of the as-is 3PL network in Day 5

To model the current network and thus the as-is scenario, we fixed the connections among the entities (i.e., suppliers to warehouses, warehouses to customers) with respect to the actual 3LP network, as presented in Tables 4.2 & 4.3. Within the 5-day horizon the company receives from its suppliers and ships products to its customers multiple times. This detailed analysis of the connectivity by day of the horizon is presented in Appendix D.

Table 4. 2. As-is general connectivity between suppliers and warehouses

To…	Suppliers 1-22	Supplier 20	Supplier 21	Supplier 23
Warehouse 1	•	•		
Warehouse 2		•		•
Warehouse 3			•	

Table 4. 3. As-is general connectivity between warehouses and customers

From…	Customers 1-48	Customers 32-33	Customers 41-44	Customers 49-53
Warehouse 1	•	•	•	
Warehouse 2		•		•
Warehouse 3			•	

Table 4. 4. As-is product allocation at each warehouse

Products at…	Product 1-37	Product 38	Product 39	Product 40	Product 41
Warehouse 1	•	•	•	•	

Warehouse 2	•		•
Warehouse 3		•	

In the as-is model we also limited the type of vehicles used to (v = 1, v = 2, v = 3) and the combinations of these vehicles: v = 5, v = 7, v = 8, v = 9 v = 10, v = 11, v = 13, v = 14, v = 15.

Table 4. 5. Capacity and inventory, receiving and shipping cost at warehouses

Warehouses	Capacity (in pallets)	Receiving cost (€)	Inventory cost (€)	Picking & shipping cost (€)
1	22000	2.4	0.216	2.4
2	8000	2.0	0.18	2.0
3	3000	1.6	0.144	1.6

Outputs of the network

For the as-is network, the model optimized only the transportation quantities for throughout the time horizon (i.e., 5 days). Specifically, the solution provided the values of the following decision variable which were not provided by the company: transportation type and quantities per day for each pre-defined connection.

In the as-is scenario, warehouse 1 manages 59.8% of the total inventory while warehouses 2 and warehouse 3 manage 36.2% and 3.8%, respectively, as presented in Table 4.6. The allocations of each product to one or more warehouses are an input for the as is scenario (see Table 4.4).

The model included 6,686 decision variables and 3,042 constraints; the run time to optimality was approximately 18,000 sec (5 hr) on the system mentioned previously.

Table 4.6. As-is inventory at each warehouse per day in pallets for all products. Day 0 inventory is the initial inventory (input)

Inventory at...	Day 0	Day 1	Day 2	Day 3	Day 4	Day 5	Pallets shipped to...	Pallets shipped from...
Warehouse 1	6,666	6,450	6,296	6,206	6,075	5,911	1,618	2,373
Warehouse 2	3,879	3,690	3,640	3,630	3,607	3,584	975	1,270
Warehouse 3	577	540	489	452	432	380	0	197
Total	**11,112**	**10,680**	**10,425**	**10,288**	**10,114**	**9,875**	**2,593**	**3,840**

Note that stock is turned every 20 days in each warehouse, according to company data.

Based on the model's results of the as-is scenario, Tables 4.7 and 4.8 below present the number of trucks delivering or shipping to/from each warehouse during each day of the under-study horizon, respectively. Again, these are model outputs.

Table 4. 7. As-is number of trucks between suppliers and warehouses per day

Trucks to…	Day 1	Day 2	Day 3	Day 4	Day 5	Total
Warehouse 1	27	11	16	16	16	86
Warehouse 2	5	9	9	7	9	39
Warehouse 3	0	0	0	0	0	0
Total	**32**	**20**	**25**	**23**	**25**	**125**

Table 4. 8. As-is number of trucks between warehouses and customers per day

Trucks from…	Day 1	Day 2	Day 3	Day 4	Day 5	Total
Warehouse 1	40	27	14	22	32	135
Warehouse 2	12	12	9	11	10	54
Warehouse 3	3	4	3	2	4	16
Total	**55**	**43**	**26**	**35**	**46**	**205**

As described in Section 4.1, the as-is scenario represents as much as possible the real status of the 3PL company for the selected horizon based on actual data provided from the 3PL company and some reasonable assumptions. More specifically, in the as-is scenario, the connectivity is totally in line with the company's actual network; thus, the proposed MILP model considers as fixed the connections among the entities. However the transportation quantities were optimized for the under-study horizon. Thus, the number of the actual trucks is higher than those specified by the model in the as-is scenario. More specifically:

- 140 actual trucks arrived at warehouses instead of the 125 as per the model's as-is scenario (reduction of 11%),
- 216 actual trucks were actually used for shipping from the warehouses instead of 205 as per the model's as-is scenario (reduction of 5%).

As a result, the actual transportation cost is 7.92% higher than the transportation cost resulting from the MILP model for the as-is scenario. The total supply chain costs, taking into account both warehousing and transportation costs, resulting from the MILP model are 5.70%, lower than actual costs, as presented in detail in Table 4.9 below. This points to an opportunity for savings, even if truck routing is not changed.

Table 4. 9.Total cost difference between actual data and as-is scenario

	Actual as-is scenario	**Optimised as-is scenario**	**% difference**
Trucks to warehouses	140	125	-10.71%
Trucks from warehouses	216	205	-5.09%
Transport costs to warehouses	36,611.19	32,444.92	-11.30%
Transport costs from warehouses	27,209.10	25,788.16	-5.22%
Total transportation cost	63,820.29	58,233.08	-7.92%
Total warehousing cost	24,632.38	24,632.38	0%
Total cost	**88,452.67**	**82,865.46**	**-5.70%**

Note that, warehousing costs remain the same, since the same quantities are received and shipped per warehouse per day in both cases.

4.3 Evaluating the to-be network

For the to-be scenario we took into account the same parameters as in the as-is scenario i.e., costs for receiving, picking and shipping, inventory, transportation; product characteristics, warehouse capacities, etc. Furthermore, we respected the actual supply chain network architecture (i.e., warehouse locations, customer locations, supplier locations, types of vehicles). We optimized all decision variables of the proposed location-inventory-routing model as presented in Chapter 3. The to-be design results that are presented in this chapter are under optimality gap of 1.30%.

Table 4.8 shows the allocation of product among warehouses. The changes with the as is case are noted with a dot. A tic mark indicates no allocation change. It is observed that all warehouses have an active role in the storage of these products, in relation to the existing as-is situation. The allocation of 65.8% of products was changed (wholly or partially). Partially means that some quantity of the product was also stored in an additional warehouse.

Table 4. 10. To-be allocation of products per warehouse (√=no change, •=change)

Products	**Warehouse 1**	**Warehouse 2**	**Warehouse 3**
Product 1	✓		•
Product 2	✓		•
Product 3			•

Product 4	✓		•
Product 5	✓		
Product 6	✓		•
Product 7	✓		
Product 8			•
Product 9			•
Product 10			•
Product 11			•
Product 12	✓		•
Product 13	✓		•
Product 14	✓		•
Product 15			•
Product 16			•
Product 17			•
Product 18	✓		•
Product 19	✓		
Product 20	✓		•
Product 21	✓		
Product 22	✓		
Product 23	✓		
Product 24	✓		
Product 25	✓		
Product 26	✓		
Product 27	✓		
Product 28	✓		
Product 29	✓		
Product 30	✓		
Product 31	✓		
Product 32		•	
Product 33		•	
Product 34		•	
Product 35		•	
Product 36		•	

Product 37	•		•
Product 38		✓	
Product 39	✓	•	✓
Product 40			•
Product 41			•

In the as-is situation warehouse 3 was only used for a few products, but in the to-be situation warehouse 3 takes an active role, and now manages a part of 50% of the products that the company handles. This happens because warehouse 3 has reduced warehousing costs compared to the other two warehouses, making it more likely to be chosen to store products. Note that warehousing costs were based on assumptions and were not provided by the company. Thus, the change may be only due to our assumptions.

The following Figures 4.7-4.11 show the to-be connections among the main entities of the 3PL company, suppliers, warehouses and customers, during each day of the under-study horizon, respectively. The main difference between the as-is and the to-be networks is that warehouse 3 now manages a higher number of products.

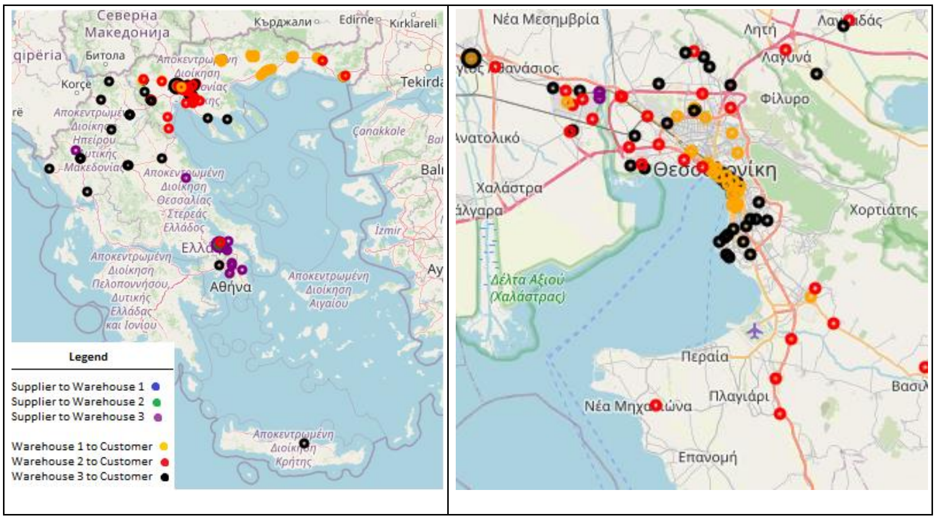

Figure 4.7. Illustration of the to-be 3PL network in Day 1

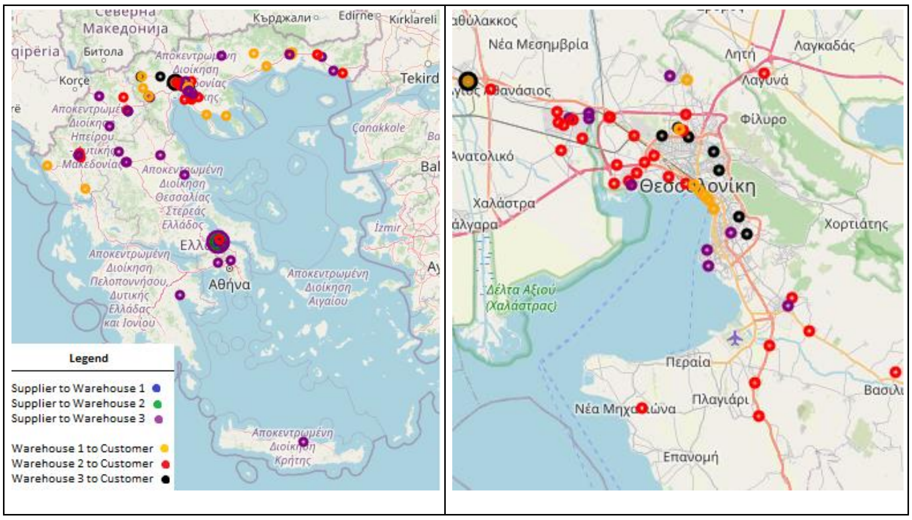

Figure 4.8. Illustration of the to-be 3PL network in Day 2

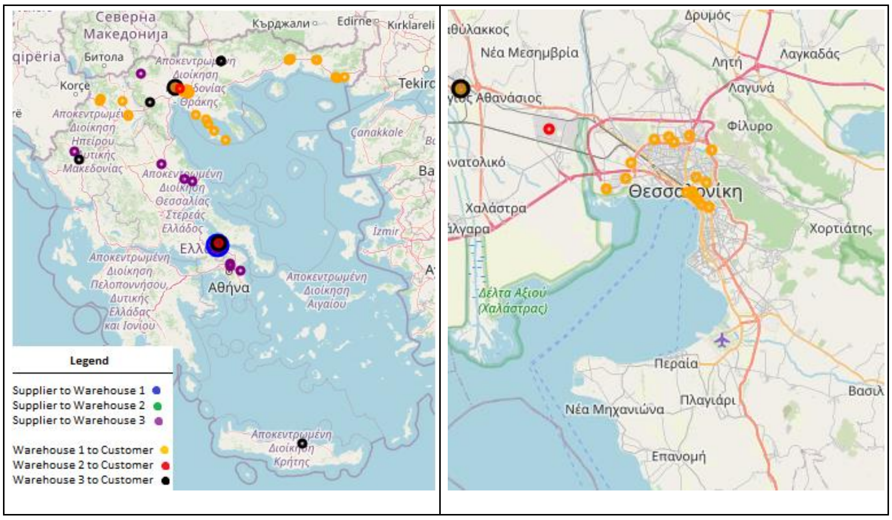

Figure 4.9. Illustration of the to-be 3PL network at Day 3

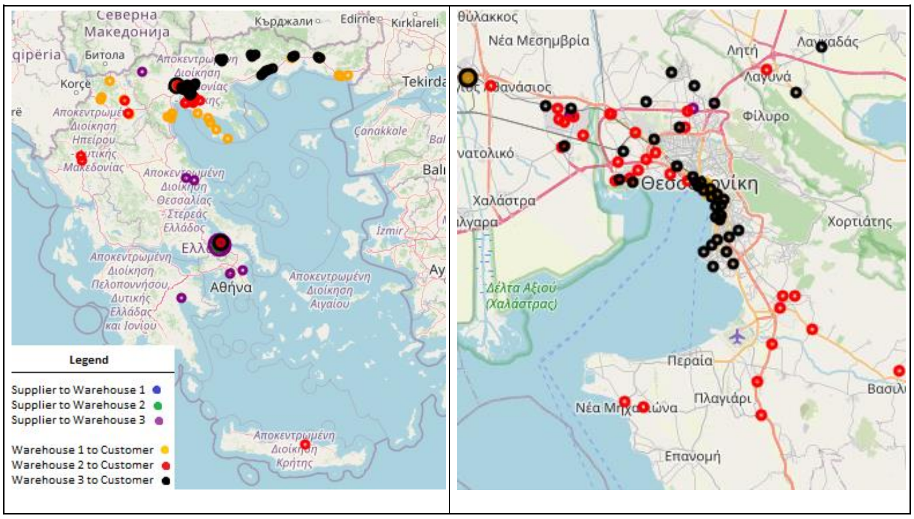

Figure 4.10. Illustration of the to-be 3PL network in Day 4

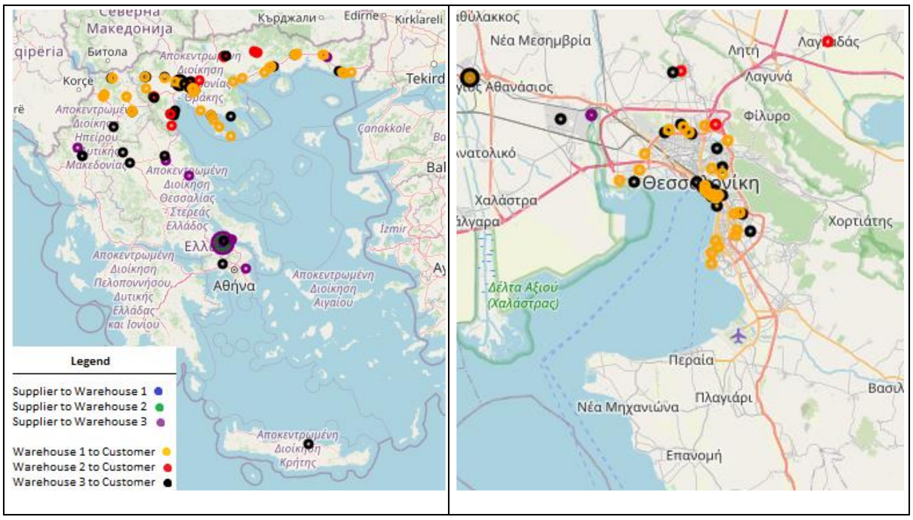

Figure 4.11. Illustration of the to-be 3PL network in Day 5

Table 4.11. To-be inventory at each warehouse per day in pallets for all product (in number of pallets). Day 0 inventory is the initial inventory (input)

Inventory at…	Day 0	Day 1	Day 2	Day 3	Day 4	Day 5	Pallets shipped for…	Pallets shipped from…
Warehouse 1	6,666	6,449	6,365	6,330	6,293	6,254	215	627
Warehouse 2	3,879	3,657	3,435	3,374	3,287	3,284	270	865
Warehouse 3	577	636	666	590	522	337	2,198	2,438
Total	**11,112**	**10,742**	**10,466**	**10,294**	**10,102**	**9,875**	**2,683**	**3,930**

Table 4.11 presents the to-be inventory at each warehouse per day. As expected, warehouse 3 handles more shipments and receipts than in the as-is scenario due to lower costs. Note that warehouse 3 has a maximum capacity is 3000 pallets, and thus may manage the related volumes.

Tables 4.12 and 4.13 below present the number of trucks that each warehouse is receiving and dispatching each day of the under-study horizon in the to-be scenario.

Table 4. 2. To-be number of trucks between suppliers and warehouses per day

Trucks to…	Day 1	Day 2	Day 3	Day 4	Day 5	Total
Warehouse 1	0	3	3	3	3	12
Warehouse 2	3	3	3	3	3	15
Warehouse 3	30	19	18	21	24	112

Total	**33**	**25**	**24**	**27**	**30**	**139**

Table 4. 3. To-be number of trucks between warehouses and customers per day

Trucks from…	Day 1	Day 2	Day 3	Day 4	Day 5	Total
Warehouse 1	11	9	7	6	10	43
Warehouse 2	13	13	3	8	4	41
Warehouse 3	27	18	12	18	30	105
Total	**51**	**40**	**22**	**32**	**44**	**189**

Table 4. 4. Trucks used between suppliers and warehouses per day for three scenarios

Scenario	Day 1	Day 2	Day 3	Day 4	Day 5	Total	% Dif vs actual as-is
Actual as-is	35	23	28	24	30	140	
As-is optimized	32	20	25	23	25	125	11.32%
To-be	33	25	24	27	30	139	-0.71%

Table 4. 5. Trucks used between warehouses and customers per day for three scenarios

Trucks to…	Day 1	Day 2	Day 3	Day 4	Day 5	Total	% Dif to actual as-is
Actual as-is	58	45	29	36	48	216	
As-is optimized	55	43	26	35	46	205	-5.22%
To-be	51	40	22	32	44	189	-13.33%

Table 4.16 compares the costs of the as is scenario (strictly based on the company inputs), the as is optimised scenario (based on the company inputs but optimising the missing variables not provided by the company as inputs), and the to be one (optimising all decision variables).

Table 4. 16. Total cost difference between as-is current, the as-is optimized and to-be scenario

	As-is current	As-is optimized	To-be scenario	% Dif As-is current & To-be	% Dif As-is opt & To-be
Trucks					
Trucks to warehouses	140	125	139	-0.7%	11.2%
Trucks from warehouses	216	205	189	-12.5%	-7.8%
Transportation trucks					
Transport costs to warehouses	36,611.1	32,444.9	31,269.8	-14,5%	-3.6%
Transport costs from warehouses	27,209.1	25,788.1	25,843.9	-5%	0.1%
Warehousing costs					
Receiving		5,817.6	4,443.2		-23.6%
Picking & Shipping		8,534.8	6,996		-18%
Inventory		10,279.9	10,312.4		0.3%
Total transportation cost	63,820.2	58,233	57,113.8	-10.5%	-1.9%
Total warehousing cost	24,632.3	24,632.3	21,751.6	-11.7%	-11.7%
Total cost	**88,452.6**	**82,865.4**	**78,865.2**	**-10.8%**	**-4.9%**

The most significant differences comparing the optimised as-is and the to-be scenarios are presented below:

- According to Table 4.10, the majority of the total to-be receipts (80.57%) and shipments (55.55%) are managed in warehouse 3, while the majority of the as-is receipts (68.80%) and shipments (65.85%) are managed in warehouse 1. The reason for this change is that warehouse 3 has lower warehousing costs compared to the other two warehouses, while the transportation cost does not have significant differences between the warehouses, since they are all located in the same area.
- According to Table 4.9, customer 1, the customer with the highest demand, is served by warehouses 1 and 3 while in the as-is scenario it is served only by warehouse 1. Supplier 23 in the as-is scenario serves only warehouse 2. In the to-be scenario Supplier 23 serves all three warehouses of the company. In the to-be scenario customers 51 (at Ritsona) and 52 (at Thessaloniki) are served by warehouses 3 and 1 respectively, for financial and geographical reasons, while in the as is scenario those customers are served only by warehouse 2.
- According to Table 4.16, the inventory cost between the optimized as-is and the to-be scenarios increases by 0.3%. This is due to the slight increase of inventory levels.

- The to-be design results to total costs of 78,865.23€, almost 11% lower to the current as-is. Extrapolating the weekly figure, network optimization may result to 500,000€ cost savings on an annual basis. The most significant difference is related to the receiving and picking / shipping costs as presented in Table 4.16. A decrease is observed for the transportation costs from node to node, that drives to a notable cost decrease on the rest warehousing costs due to the optimized product allocation.

Chapter 5. Conclusions and recommendations for future research

5.1 Work performed and conclusions

In this thesis we focused on the location-inventory-routing problem related to Third Party Logistics (3PL) companies. We developed a new mathematical model that addresses most significant decisions involved in efficient re-design of the related supply chain network considering multiplicity of time periods, products, warehouses, customers, suppliers, transportation types. The model is able to support the designer to make the following decisions for each time period of the planning horizon that involve selection of: (a) the warehouse(s) to store each product (SKU), (b) the inventory level per SKU per warehouse, (c) the warehouse(s) to serve each customer and (d) the appropriate vehicles to transport the products from the suppliers to the warehouses, and from the latter to the final customers.

In order to validate the consistency and correctness of the proposed MILP model, a series of validation tests have been performed. The expected solutions were obtained by the model in all cases.

Also, we applied the proposed location-inventory-routing model to a complex industrial case in order to examine the model's effectiveness. Specifically, we have investigated the re-design of the Northern Greece supply chain network of a large 3PL company operating throughout Greece. Based on the data we obtained from the company, we applied the MILP model to estimate aspects of the network operation not provided to us by the company, but respecting the basic network. In this step, we partially optimised operations (through the free part of the model), resulting in 5,70% cost savings. Subsequently we re-designed the network resulting in 10,84% overall savings.

In the optimised (to-be) model significant changes were observed in terms of:

- Allocation of product to warehouses,
- The selection of the appropriate supplier-warehouse-customer route and
- The selection of the appropriate combination of vehicles for supply and distribution.

5.2 Recommendations for future research

Future research in this supply chain network may explore some new interesting areas, as follows:

Multi-objective considerations in the supply chain network design

In addition to minimizing costs, the model could be extended to also deal with emissions, in order to promote sustainability of 3PL operations. In this way the 3PL brand and business may be enhanced.

Addition of reverse logistics to the supply chain

The model could evolve and become more complete, by considering the reverse logistics of the supply chain, which may include the returns of expired or non-conforming products by customers. We have formulated the mathematical model to serve the reverse logistics of the company, but this part was not implemented in ode. Implementation, testing and validation is an interesting extension for further study.

Extension of the existing model to consider the network design problem

The current mathematical model considers the case of redesigning an existing network with fixed warehouse locations. If the number of warehouses, their size and their location are to be selected, then appropriate extensions should be made; for example new binary variables should be defined for selecting the locations from a finite set of network nodes, appropriate constraints should secure consistent flows from/to these warehouses, etc.

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Appendices

Appendix A: Questionnaire used to obtain information about the operations of the company.

In this appendix, Table A.1 presents the questions that were needed to provide important information in the context of the research for the specific 3PL company.

Table A. 1. Table of the questionnaire we used for company

Number	Questions
1	The orders of products that arrive at the warehouses and concern the customer 1 (significant customer with many orders per week), by whom are they written?
2	When a shipment is made by the suppliers supplying the customer 1 to the warehouses and by what criteria? Strictly based on daily orders? Is there a grouping of orders? If so, by what criteria? Another method?
3	The products of the suppliers how many calendar days approximately remain stored in warehouses until their shipment to the customers?
4	Is there a forecast and if so, for how long?
5	By what criteria are customer products stored in the three different warehouses? How is the warehouse selected per supplier and per product?
6	In terms of shipping products from suppliers, when the supply is made and by what criteria? To cover safety stock? To deliver immediate to end customers?
7	Before shipments from suppliers to warehouses take place, the agreement is guided by the presence of empty pallet places on the racks? They are shipped without the information about whether there are empty pallet places in warehouses? There is a continuous flow agreement whether empty or not Pallet places on racks?
8	Orders for end customers (recipients of shipments) from who do they come from? From the Suppliers? Regular shipment from warehouses based on long-term order?
9	How often are these orders placed? Is forecast used by the company?
10	There are a minimum number of pallets for a truck to load? There is a lower bound for the vehicles?

11	The weekly demand of customer 1 but also of other customers, remains at about the same levels throughout the year? Is there a significant increase in demand due to seasonality?

Appendix B: Warehouse capacities and warehousing costs.

In this appendix we present in Table B.1 the capacities of the warehouses of the company and the warehousing cost of each.

Table B. 1. Table of warehouse capacities and warehousing costs

Warehouses	Warehouse capacity in pallets	Receiving cost (€)	Storage cost (€)	Picking & Shipping cost (€)
1	22000	2.4	0.216	2.4
2	8000	2	0.18	2
3	3000	1.6	0.144	1.6

Appendix C: Transportation costs.

In this appendix we present in Table C.1 the transportation costs per vehicle and their analysis.

Table C. 1. Table of transportation costs analysis per vehicle

Vehicle type	Vehicle capacity in pallets	Driver cost (€)	Cost per km (€)	Depreciation per km (€)	Min cost of route (€)
1	7	30	0.11	0.022	50
2	18	40	0.275	0.055	50
3	33	100	0.6	0.16	50

Appendix D: As-is connectivity.

In this appendix we present in Table D.1 the as-is connectivity between suppliers and warehouses per day and in Table D.2 the as-is connectivity between warehouses and customers.

Table D. 1. As-is connectivity between suppliers and warehouses (The values in each cell represent the warehouse(s) the related supplier visits the related day)

	Days				
Suppliers	1	2	3	4	3

1	Warehouse 1				
2	Warehouse 1	Warehouse 1	Warehouse 1		
3		Warehouse 1		Warehouse 1	Warehouse 1
4	Warehouse 1	Warehouse 1			Warehouse 1
5	Warehouse 1		Warehouse 1		Warehouse 1
6			Warehouse 1	Warehouse 1	
7	Warehouse 1		Warehouse 1		
8	Warehouse 1				Warehouse 1
9	Warehouse 1		Warehouse 1	Warehouse 1	Warehouse 1
10	Warehouse 1	Warehouse 1	Warehouse 1	Warehouse 1	
11	Warehouse 1		Warehouse 1		Warehouse 1
12		Warehouse 1		Warehouse 1	
13	Warehouse 1	Warehouse 1	Warehouse 1	Warehouse 1	Warehouse 1
14	Warehouse 1				
15		Warehouse 1			Warehouse 1
16				Warehouse 1	
17	Warehouse 1	Warehouse 1			
18	Warehouse 1	Warehouse 1			Warehouse 1
19			Warehouse 1	Warehouse 1	
20					
21					
22	Warehouse 1			Warehouse 1	
23	Warehouse 2	Warehouse 2	Warehouse 2	Warehouse 2	Warehouse 2

From the table above it may be seen, that supplier 23 ships products to warehouse 2 daily, and so does supplier 13 to warehouse 1. Supplier 10 ships 4 days out of 5 of the time horizon.

Table D. 2. As-is connectivity between warehouses and customers (The values in each cell represent the warehouse(s) the related supplier visits the related day)

	Days				
Customers	**1**	**2**	**3**	**4**	**3**
1	Warehouse 1	Warehouse 1	Warehouse 1	Warehouse 1	Warehouse 1
2	Warehouse 1	Warehouse 1			

3	Warehouse 1	Warehouse 1			
4	Warehouse 1	Warehouse 1			
5	Warehouse 1	Warehouse 1			
6	Warehouse 1	Warehouse 1			
7	Warehouse 1	Warehouse 1			
8	Warehouse 1	Warehouse 1			Warehouse 1
9	Warehouse 1	Warehouse 1			Warehouse 1
10	Warehouse 1	Warehouse 1			Warehouse 1
11	Warehouse 1				Warehouse 1
12	Warehouse 1	Warehouse 1			Warehouse 1
13	Warehouse 1	Warehouse 1			Warehouse 1
14	Warehouse 1	Warehouse 1			Warehouse 1
15	Warehouse 1				Warehouse 1
16	Warehouse 1	Warehouse 1			
17	Warehouse 1	Warehouse 1			Warehouse 1
18	Warehouse 1	Warehouse 1			Warehouse 1
19	Warehouse 1				Warehouse 1
20	Warehouse 1				
21					Warehouse 1
22			Warehouse 1	Warehouse 1	Warehouse 1
23				Warehouse 1	Warehouse 1
24					Warehouse 1
25			Warehouse 1		Warehouse 1
26				Warehouse 1	Warehouse 1
27				Warehouse 1	Warehouse 1
28					Warehouse 1
29			Warehouse 1	Warehouse 1	Warehouse 1
30					Warehouse 1
31			Warehouse 1	Warehouse 1	Warehouse 1
32		Warehouse 2		Warehouse 2	
33		Warehouse 2			Warehouse 2
34	Warehouse 1				
35	Warehouse 1				Warehouse 1

36		Warehouse 1		Warehouse 1	
37				Warehouse 1	
38					Warehouse 1
39	Warehouse 1	Warehouse 1			
40	Warehouse 1	Warehouse 1		Warehouse 1	Warehouse 1
41	Warehouse 3	Warehouse 3	Warehouse 3	Warehouse 3	Warehouse 3
42	Warehouse 3	Warehouse 3		Warehouse 3	Warehouse 3
43	Warehouse 3	Warehouse 3	Warehouse 3		Warehouse 3
44		Warehouse 3			Warehouse 3
45	Warehouse 1			Warehouse 1	
46	Warehouse 1			Warehouse 1	
47	Warehouse 1			Warehouse 1	
48	Warehouse 1			Warehouse 1	
49	Warehouse 2	Warehouse 2		Warehouse 2	
50	Warehouse 2				
51	Warehouse 2	Warehouse 2	Warehouse 2	Warehouse 2	Warehouse 2
52	Warehouse 2	Warehouse 2	Warehouse 2	Warehouse 2	Warehouse 2
53	Warehouse 2	Warehouse 2	Warehouse 2		Warehouse 2

Customer 1 has daily demand and is served by warehouse 1. Also, customers 43 and 52 equally have daily demand that is met by warehouses 3 and 2 respectively. It may also be seen that several customers demand products once or more times per week, and are served by the warehouses.